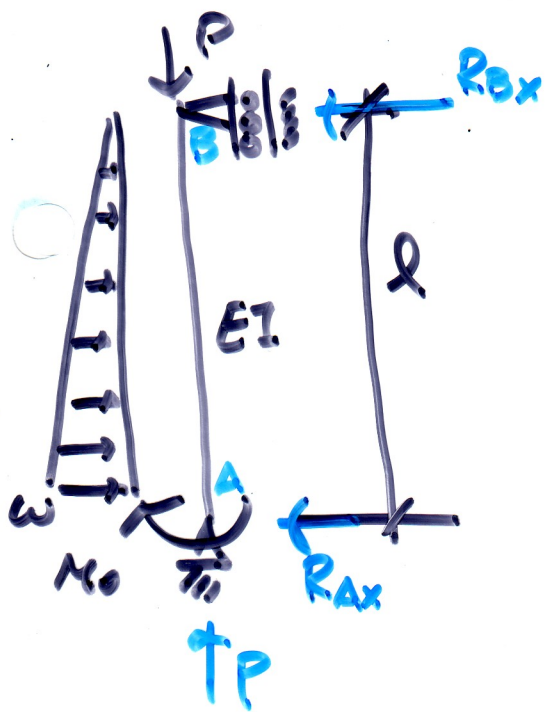
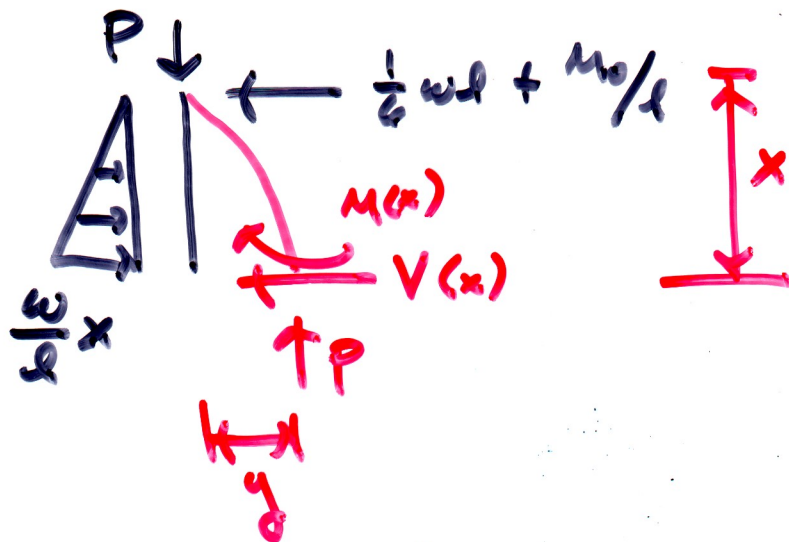




$$\sum M_A = 0$$

$$R_{Bx} l = \frac{1}{2} w l \cdot \frac{l}{3} + M_0$$

$$R_{Bx} = \frac{1}{6} w l + \frac{M_0}{l}$$



$$M(x) + \frac{1}{2} \frac{w}{l} x \cdot x \cdot \frac{x}{3} = P y + \left( \frac{1}{6} w l + \frac{M_0}{l} \right) x$$

$$EI \frac{d^2 y}{dx^2} + P y = \frac{1}{6} \frac{w}{l} x^3 - \left( \frac{1}{6} w l + \frac{M_0}{l} \right) x$$

$$\frac{d^2 y}{dx^2} + K^2 y = \frac{1}{6} \frac{w}{EI l} x^3 - \frac{1}{EI} \left( \frac{1}{6} w l + \frac{M_0}{l} \right) x$$

$$\hookrightarrow y_h = C_1 \sin kx + C_2 \cos kx$$

$$y_p = C_3 x^4 + C_4 x^3 + C_5 x^2 + C_6 x + C_7$$

$$\hookrightarrow \frac{dy}{dx} = 4C_3 x^3 + 3C_4 x^2 + 2C_5 x + C_6$$

$$\frac{d^2 y}{dx^2} = 12C_3 x^2 + 6C_4 x + 2C_5$$

$$12 C_3 x^2 + 6 C_4 x + 2 C_5 + k^2 (C_3 x^4 + C_4 x^3 + C_5 x^2 + C_6 x + C_7) =$$

$$\frac{1}{6} \frac{\omega}{EI l} x^3 - \frac{1}{EI} \left( \frac{1}{6} \omega l + \frac{M_0}{l} \right) x$$

$$\hookrightarrow C_3 = 0$$

$$\hookrightarrow \frac{P}{EI} C_4 = \frac{1}{6} \frac{\omega}{EI l} \Rightarrow C_4 = \frac{1}{6} \frac{\omega}{Pl}$$

$$\hookrightarrow C_5 = 0$$

$$\hookrightarrow 6 \left( \frac{1}{6} \frac{\omega}{Pl} \right) + \frac{P}{EI} C_6 = -\frac{1}{EI} \left( \frac{1}{6} \omega l + \frac{M_0}{l} \right)$$

$$C_6 = -\frac{\omega l}{6P} - \frac{M_0}{Pl} - \frac{\omega EI}{P^2 l}$$

$$\hookrightarrow C_7 = 0$$

$$\Rightarrow y = C_1 \sin kx + C_2 \cos kx + \frac{1}{6} \frac{\omega}{Pl} x^3 - \left( \frac{\omega l}{6P} + \frac{M_0}{Pl} + \frac{\omega EI}{P^2 l} \right) x$$

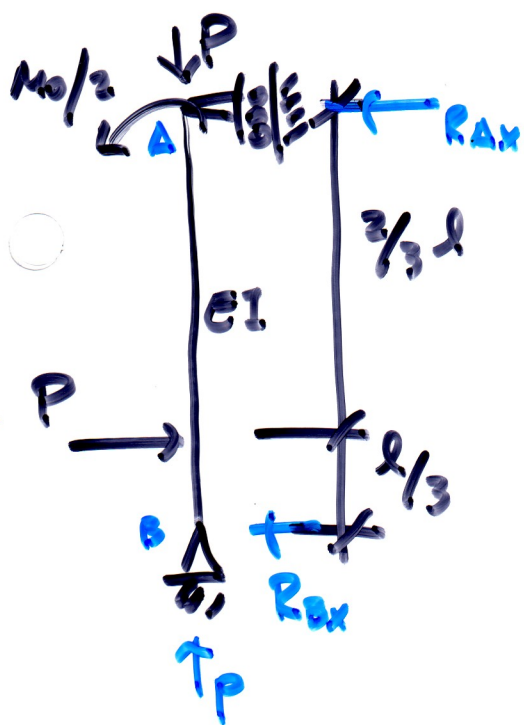
$$\hookrightarrow \text{Condiciones: } 1) x=0, y=0 \quad 2) x=l, y=0$$

$$1) 0 = C_2$$

$$2) 0 = C_1 \sin kl + \frac{1}{6} \frac{\omega}{Pl} l^3 - \left( \frac{\omega l}{6P} + \frac{M_0}{Pl} + \frac{\omega EI}{P^2 l} \right) l$$

$$\hookrightarrow C_1 = \frac{M_0}{P \sin kl} + \frac{\omega EI}{P^2 \sin kl}$$

$$\Rightarrow y = \left( \frac{M_0}{P \sin kl} + \frac{\omega EI}{P^2 \sin kl} \right) \sin kx + \frac{1}{6} \frac{\omega}{Pl} x^3 - \left( \frac{\omega l}{6P} + \frac{M_0}{Pl} + \frac{\omega EI}{P^2 l} \right) x$$

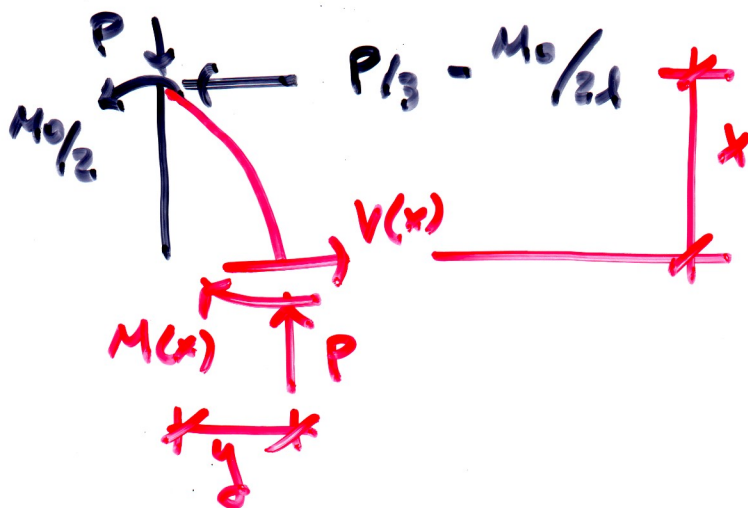


Encuentre  $y = f(x)$  para  $0 < x \leq \frac{2}{3}l$

$$\sum M_0 = 0$$

$$R_{Ax} \cdot l = P \cdot \frac{l}{3} - \frac{M_0}{2}$$

$$R_{Ax} = \frac{P}{3} - \frac{M_0}{2l}$$



$$M(x) = Py + \left( \frac{P}{3} - \frac{M_0}{2l} \right) x + \frac{M_0}{2}$$

$$EI \frac{d^2 y}{dx^2} + Py = -\frac{M_0}{2} - \left( \frac{P}{3} - \frac{M_0}{2l} \right) x$$

$$\frac{d^2 y}{dx^2} + k^2 y = -\frac{M_0}{2EI} - \frac{1}{EI} \left( \frac{P}{3} - \frac{M_0}{2l} \right) x$$

$$\hookrightarrow y_h = C_1 \sin kx + C_2 \cos kx$$

$$y_p = C_3 x^2 + C_4 x + C_5$$

$$\hookrightarrow \frac{dy}{dx} = 2C_3 x + C_4$$

$$\frac{d^2 y}{dx^2} = 2C_3$$

$$2C_3 + \frac{P}{EI} (C_3 x^2 + C_4 x + C_5) = -\frac{M_0}{2EI} - \frac{1}{EI} \left( \frac{P}{3} - \frac{M_0}{2l} \right) x$$

$$\hookrightarrow C_3 = 0$$

$$\hookrightarrow \frac{P}{EI} C_4 = -\frac{1}{EI} \left( \frac{P}{3} - \frac{M_0}{2l} \right) \Rightarrow C_4 = -\frac{1}{3} + \frac{M_0}{2Pl}$$

$$\hookrightarrow \frac{P}{EI} C_5 = -\frac{M_0}{2EI} \Rightarrow C_5 = -\frac{M_0}{2P}$$

$$\Rightarrow y = C_1 \sin Kx + C_2 \cos Kx - \left( \frac{1}{3} - \frac{M_0}{2Pl} \right) x - \frac{M_0}{2P}$$

$$\hookrightarrow \text{Condiciones: 1) } x=0, y=0, \text{ 2) } x=0, V(x) = \frac{P}{3} - \frac{M_0}{2l}$$

$$1) 0 = C_2 - \frac{M_0}{2P} \Rightarrow C_2 = \frac{M_0}{2P}$$

$$\Rightarrow y = C_1 \sin Kx + \frac{M_0}{2P} \cos Kx - \left( \frac{1}{3} - \frac{M_0}{2Pl} \right) x - \frac{M_0}{2P}$$

$$y' = C_1 K \cos Kx - \frac{M_0}{2P} K \sin Kx - \left( \frac{1}{3} - \frac{M_0}{2Pl} \right)$$

$$y'' = -C_1 K^2 \sin Kx - \frac{M_0}{2P} K^2 \cos Kx$$

$$y''' = -C_1 K^3 \cos Kx + \frac{M_0}{2P} K^3 \sin Kx$$

$$\Rightarrow -EI \left( -C_1 K^3 \cos Kx + \frac{M_0}{2P} K^3 \sin Kx \right) - P \left( C_1 K \cos Kx - \frac{M_0}{2P} K \sin Kx - \frac{1}{3} + \frac{M_0}{2Pl} \right) = \frac{P}{3} - \frac{M_0}{2l}$$

$$2) C_1 EI K^3 - C_1 PK + P/3 - M_0/2l = P/3 - M_0/2l \leadsto C_1 = 0$$

$$\Rightarrow y = \frac{M_0}{2P} \cos Kx - \left( \frac{1}{3} - \frac{M_0}{2Pl} \right) x - \frac{M_0}{2P}$$

## Longitud Efectiva para las columnas de un marco

K depende de la rigidez relativa de las columnas con respecto a las vigas que llegan a cada nudo ( $G$ ).

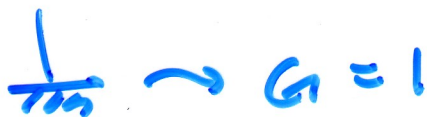
$$G = \frac{\sum E_c I_c / L_c}{\sum E_g I_g / L_g} = \frac{\sum I_c / L_c}{\sum I_g / L_g} \left\{ \begin{array}{l} \text{columnas} \\ \text{vigas} \end{array} \right.$$

$$E_c = E_g = E = 29,000 \text{ ksi}$$

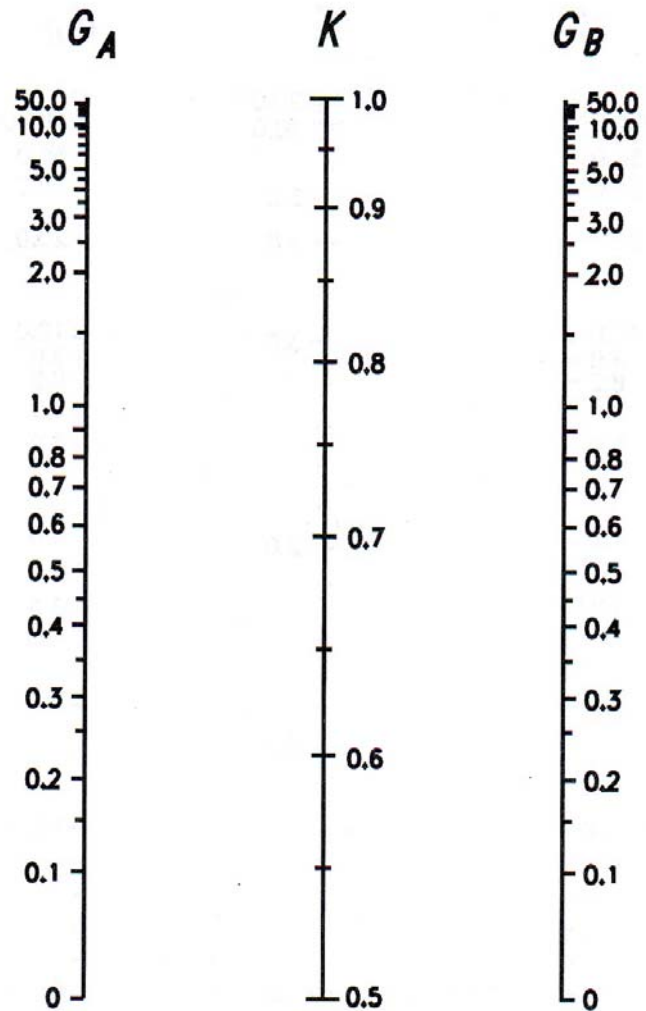


$\Rightarrow$  Con los valores de  $G_A$  y  $G_B$  se entra en los nomogramas de Jackson-Moreland y se determina K gráficamente.

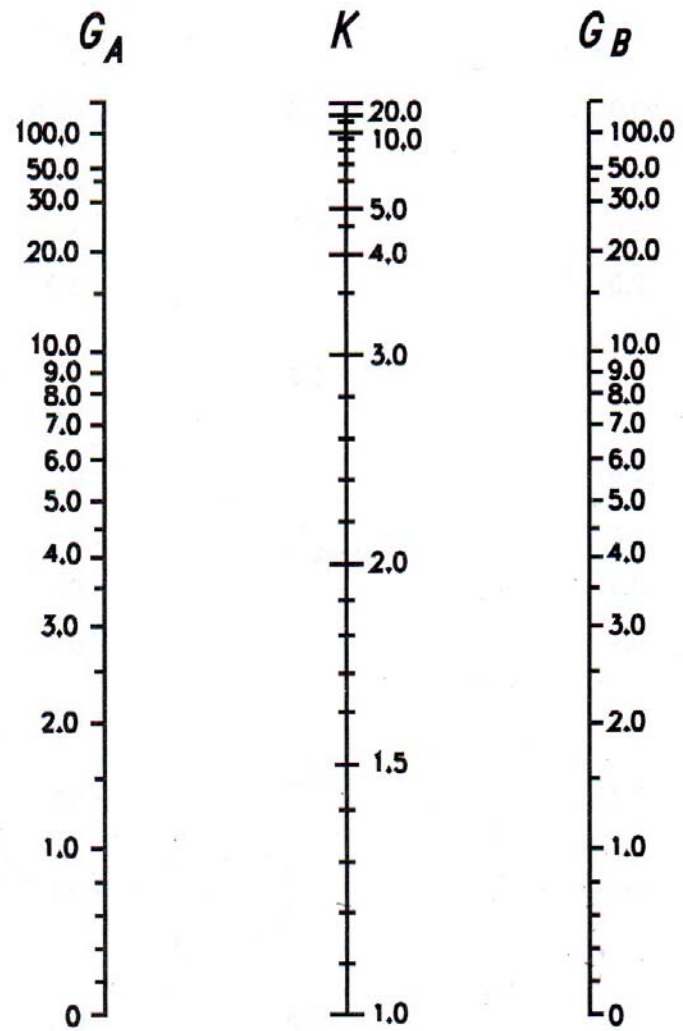
Para las columnas de Planta Baja:



Nota:  $0.5 \leq K \leq 1.0 \leadsto$  Marcos Avriostados Lateralmente  
 $1.0 < K < \infty \leadsto$   $\checkmark$  No  $\checkmark$   $\checkmark$



SIDESWAY INHIBITED



SIDESWAY UNINHIBITED