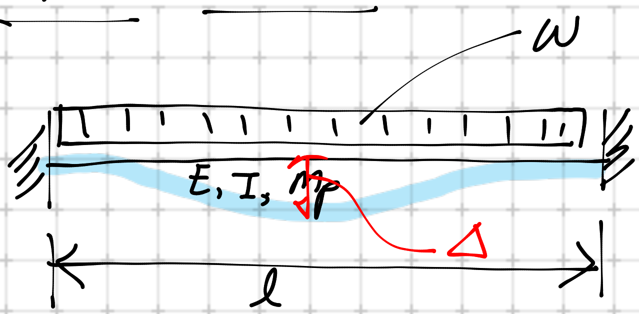
14/Abr/2014

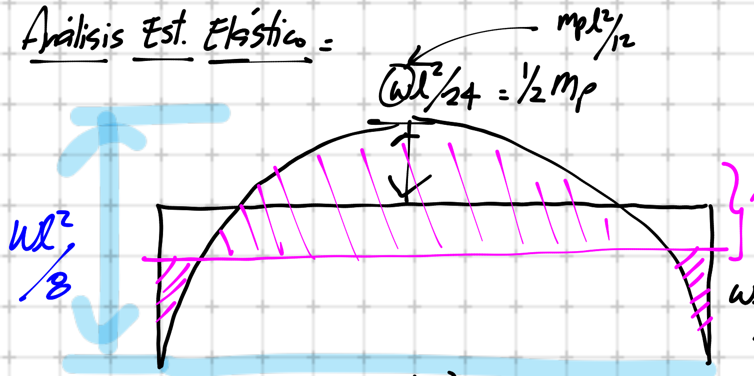
Análisis Plástico



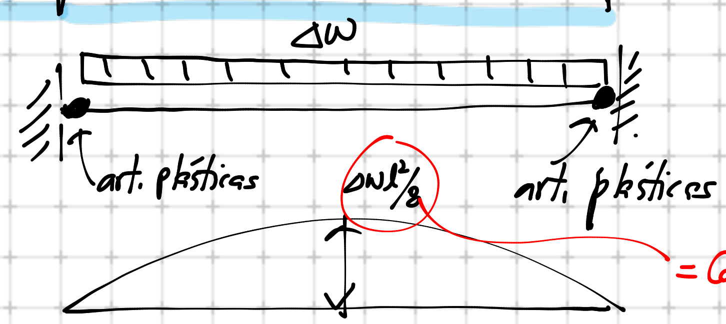
$M_p = \sigma_y Z$

Método Paso a Paso:

Análisis Est. Elástico:

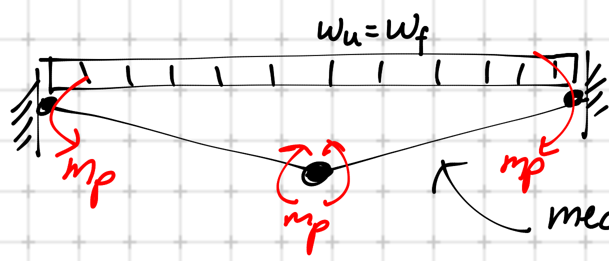


$\frac{wl^2}{12} = M_p \rightsquigarrow w = \frac{12M_p}{l^2}$



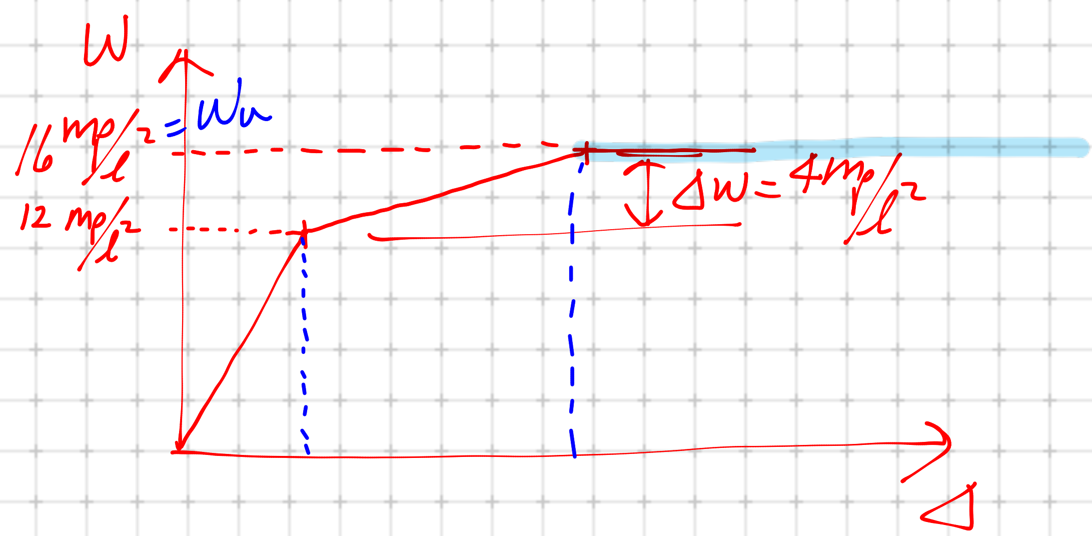
= Capacidad Restante $= \frac{1}{2} M_p$

$\frac{\Delta W l^2}{8} = \frac{1}{2} M_p \rightsquigarrow \Delta W = \frac{4M_p}{l^2}$

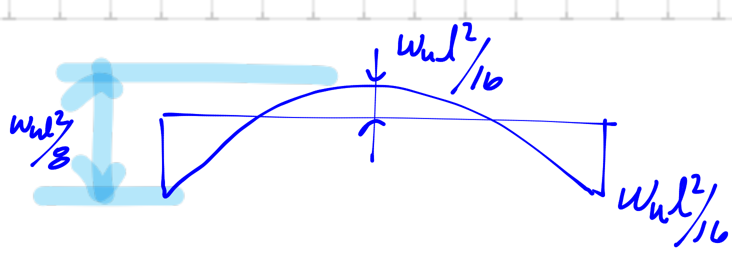


$$\Rightarrow W_u = W + \Delta W = \frac{12M_p}{l^2} + 4\frac{M_p}{l^2} \rightsquigarrow \boxed{W_u = \frac{16M_p}{l^2}}$$

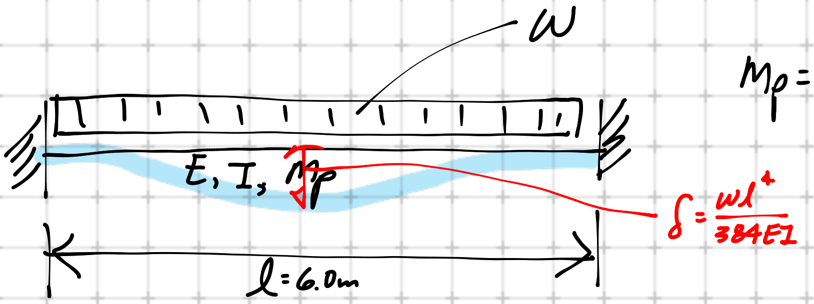
$$\boxed{M_p = \frac{W_u l^2}{16}}$$



Análisis Plástico:



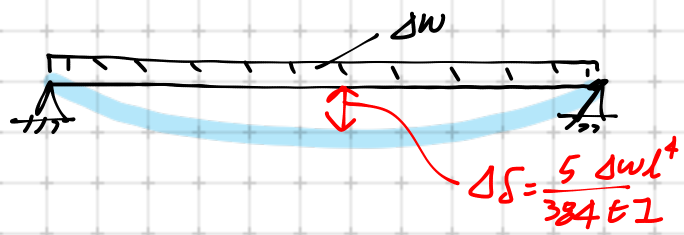
Ejemplo: Viga de acero W21x50: $\begin{cases} I = 984\ in^4 = 410\times10^{-6}\ m^4 \\ Z = 110\ in^3 = 1.80\times10^{-3}\ m^3 \\ \sigma_y = 350\ MPa \\ E = 200\ GPa \end{cases}$ → - Carga última que soporta ($w_u = ?$)
- Gráfica w vs δ



$M_p = \sigma_y Z = (350\times10^6)(1.80\times10^{-3}) = 630\ kN\text{-}m$

↳ Primera art. plástica: $w = 12M_p/l^2 = 210\ kN/m$

$\delta = \frac{wl^4}{384EI} = 8.64\ mm$

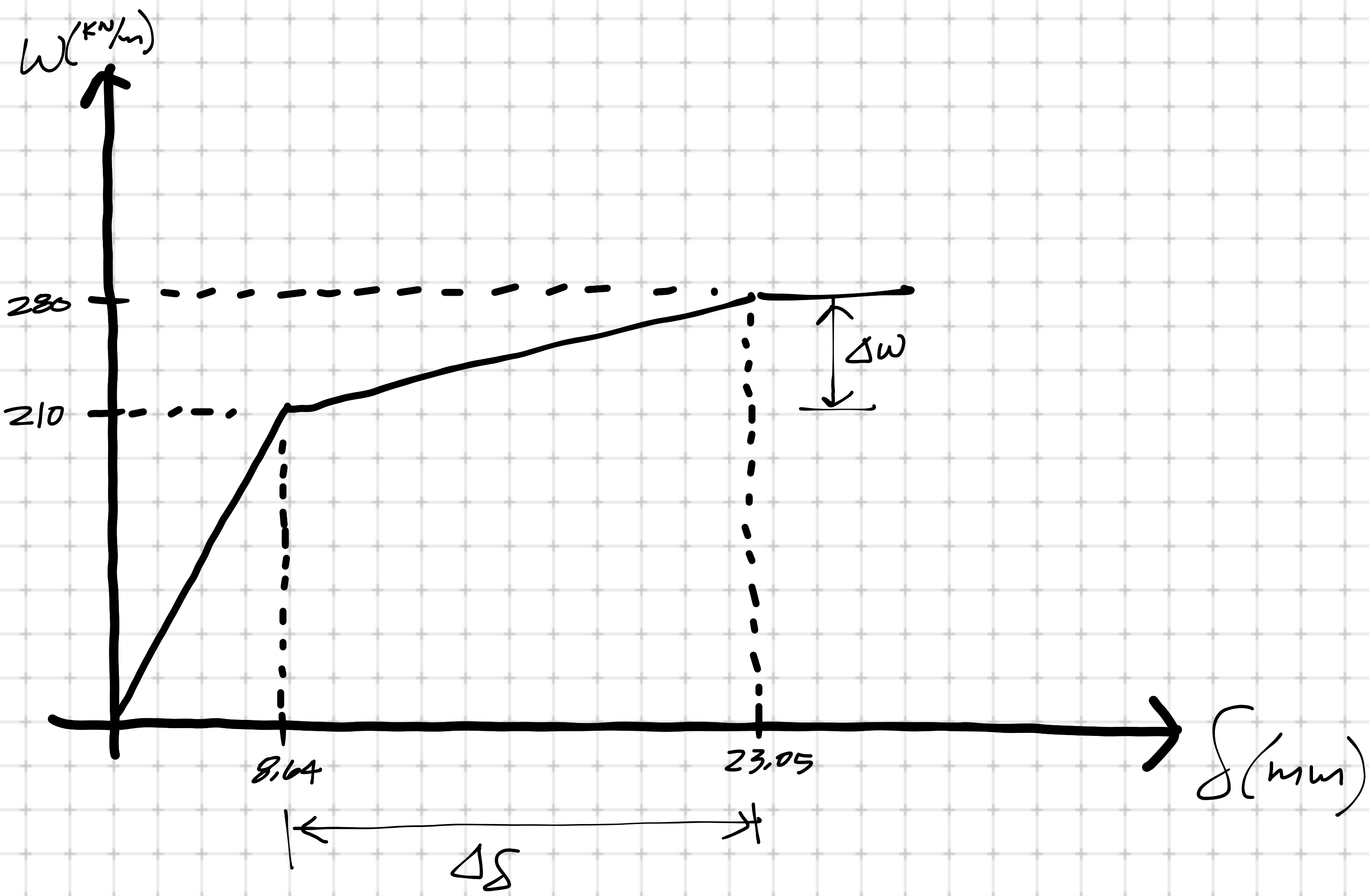


↳ Segunda art. plástica: $\Delta w = 4M_p/l^2 = 70\ kN/m$

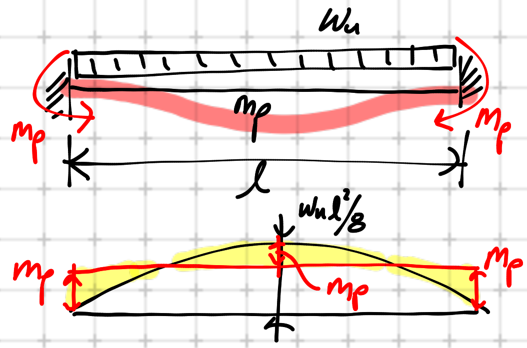
$\Delta\delta = \frac{5}{384}\frac{\Delta w l^4}{EI} = 14.41\ mm$

$W_u = W + \Delta W = 210 + 70 = 280\ kN/m$

$\delta_u = \delta + \Delta\delta = 8.64 + 14.41 = 23.05\ mm$



Método de Equilibrio

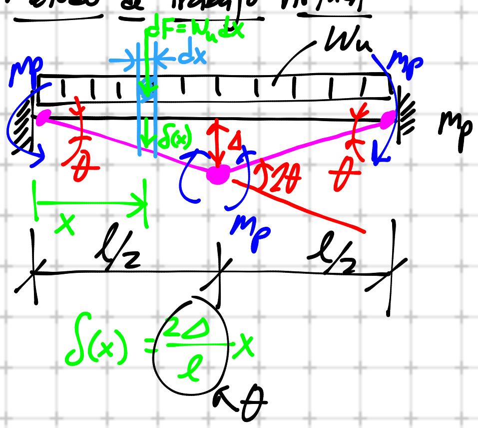


$$M_p + M_p = \frac{w_u l^2}{8} \sim \boxed{M_p = \frac{w_u l^2}{16}} \to \boxed{W_u = \frac{16 M_p}{l^2}} \to \boxed{W_u = 280 \text{ kN/m}}$$

$\to 630 \text{ kN-m}$

$\to 6{,}0 \text{ m}$

Método de Trabajo Virtual



$$W_{ext} = W_{int}$$

$$dW_{ext} = dF \cdot \delta(x)$$

$$W_{ext} = 2\int_0^{l/2} W_u \, dx \cdot \frac{2\Delta}{l} \cdot x = \overset{2}{\cancel{4}} \frac{W_u \cdot \Delta}{l} \cdot \frac{x^2}{\cancel{2}} \Bigg|_0^{l/2} = \overbrace{W_u\left[\frac{1}{2} \cdot l \cdot \Delta\right]}^{\text{área del mec. de colapso}}$$

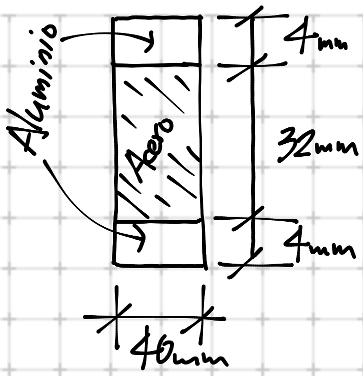
$$W_{int} = \sum_{i=1}^{n} M_{p_i} \cdot \theta_i = M_p \theta + M_p(2\theta) + M_p \theta = 4M_p\left(\frac{2\Delta}{l}\right) = 8\frac{M_p \Delta}{l}$$

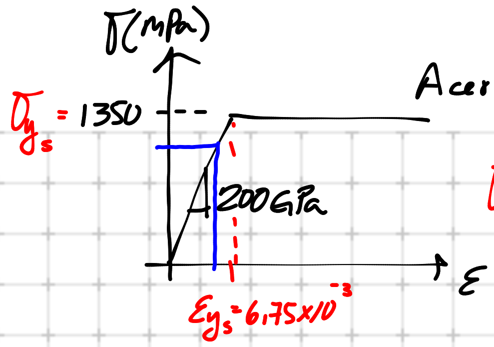
$$W_u \frac{1}{2} l \cancel{\Delta} = 8 \frac{M_p \cancel{\Delta}}{l}$$

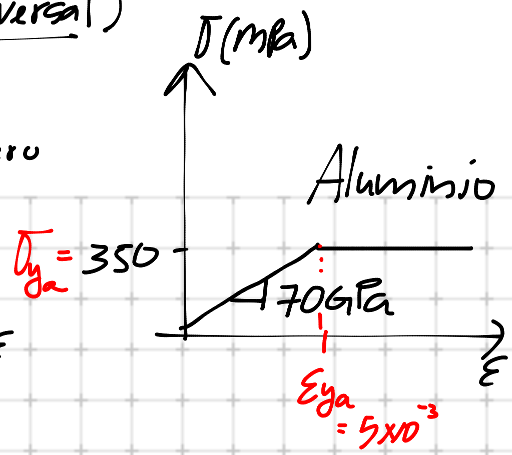
$$\boxed{W_u = \frac{16 M_p}{l^2}}$$

Análisis Plástico (A nivel de Sección transversal)

16/Abr/2014





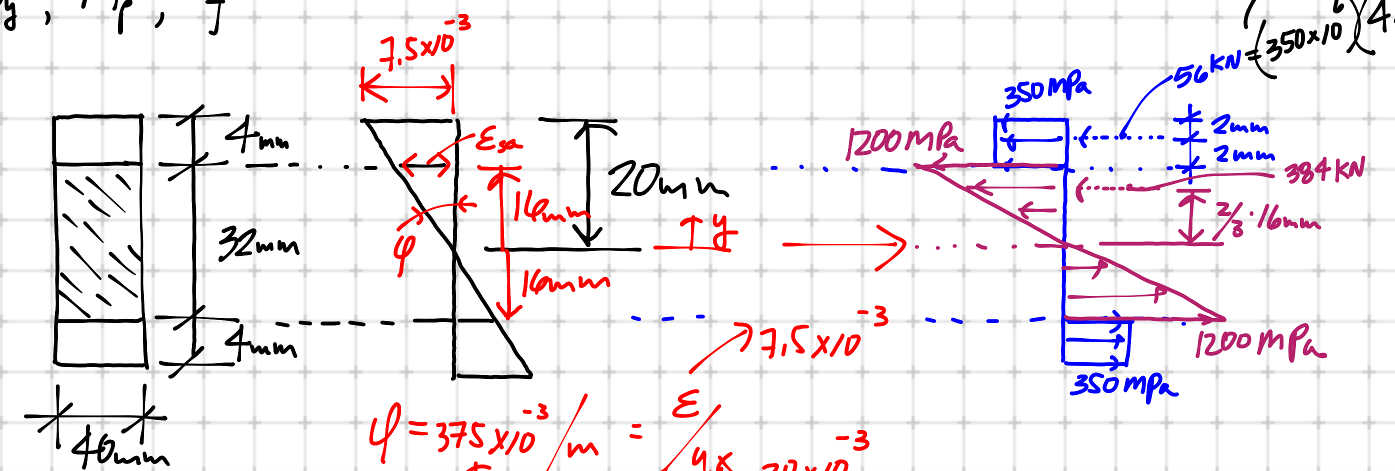


– Si $\varepsilon = 7.5\times10^{-3}$ en la fibra superior $\rightsquigarrow$ M aplicado

– M_y, M_p, f

Vol. de la dist. de esf.

$$56\,\text{kN} = \underbrace{(350\times10^{6})(4\times10^{-3})}_{\text{Área del Diag. de } \sigma}\underbrace{(40\times10^{-3})}_{\text{Ancho de la Viga}}$$



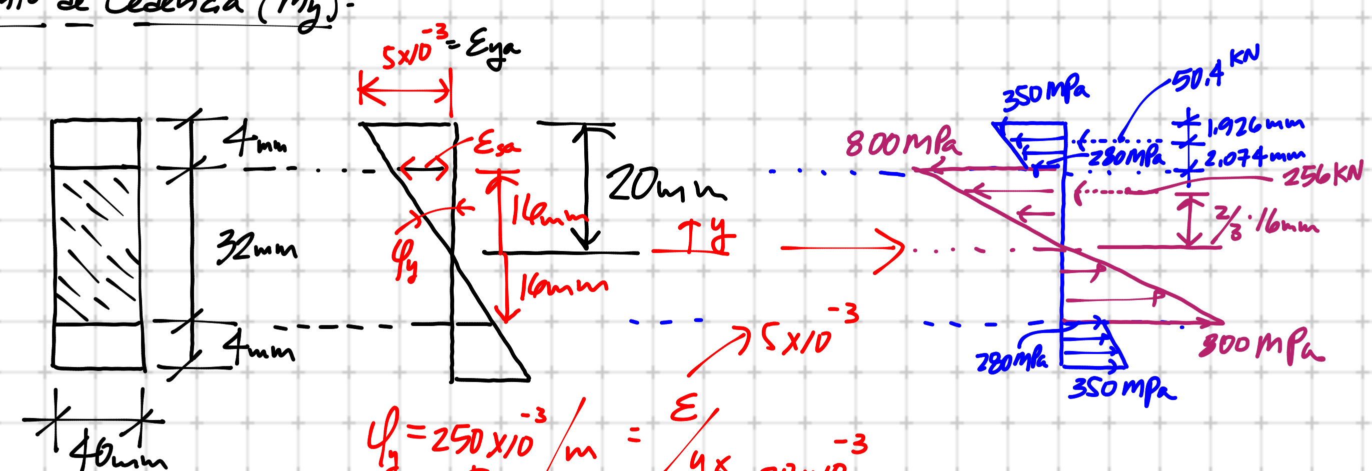
$$\varphi = 375\times10^{-3}/m = \frac{\varepsilon}{y} \quad (\varepsilon = 7.5\times10^{-3},\ y = 20\times10^{-3})$$

$$\varepsilon_{sa} = \varphi \times y = 6\times10^{-3} \quad (y = 16\times10^{-3})$$

$$\hookrightarrow \sigma_s = \varepsilon_{sa} \times E_s = 1200\,\text{MPa} \quad (E_s = 200\,\text{GPa})$$

$$\Rightarrow M = 2\times\left[56\times(18\times10^{-3}) + 384\left(\tfrac{2}{3}\cdot16\times10^{-3}\right)\right] \Rightarrow \boxed{M = 10.208\,\text{kN-m}} \rightsquigarrow \varphi = \frac{\varepsilon}{y} = \frac{7.5\times10^{-3}}{20\times10^{-3}} \Rightarrow \boxed{\varphi = 0{,}375/m}$$

Momento de Cedencia (M_y):



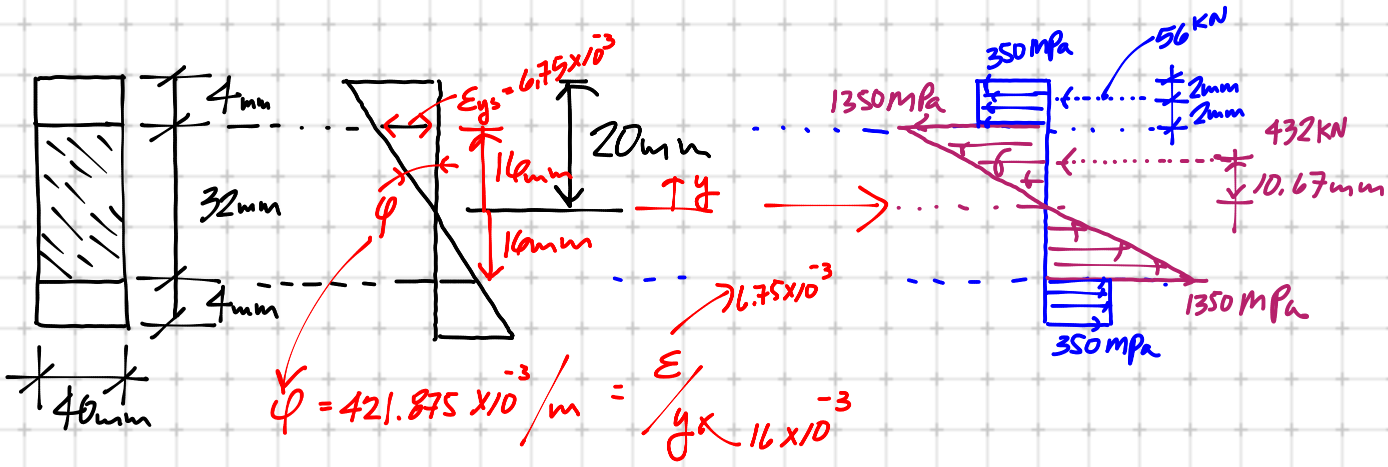
$$\varphi_y = 250\times10^{-3}/m = \frac{\varepsilon}{y} \quad (y = 20\times10^{-3})$$

$$\varepsilon_{sa} = \varphi \times y = 4\times10^{-3} \quad (y = 16\times10^{-3})$$

$$\hookrightarrow \sigma_s = \varepsilon_{sa} \times E_s = 800\,\text{MPa} \ (E_s = 200\,\text{GPa}) \ ; \ \sigma_a = \varepsilon_{sa}\, E_a = 280\,\text{MPa} \ (\varepsilon_{sa} = 4\times10^{-3},\ E_a = 70\,\text{GPa})$$

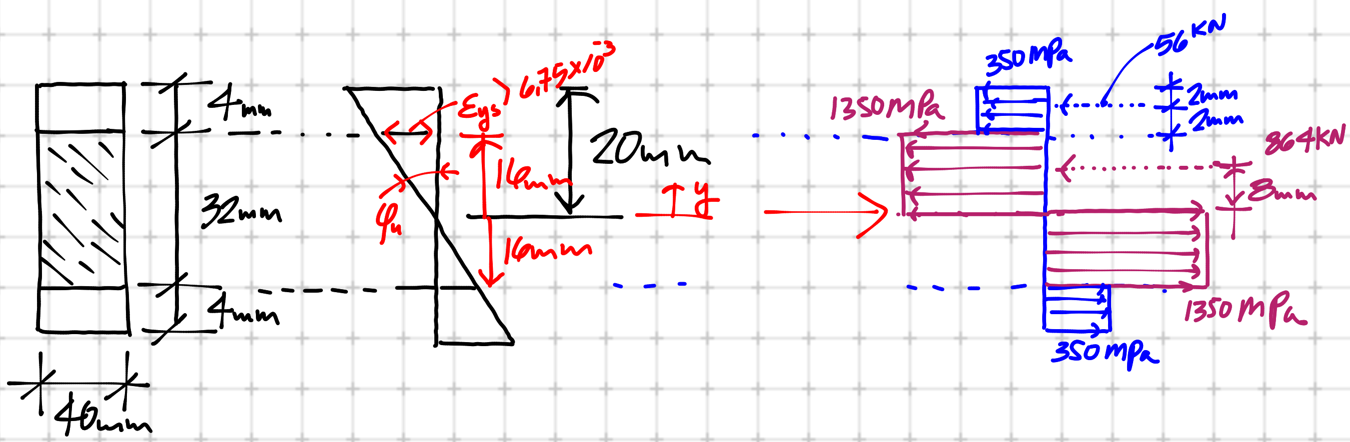
$$\Rightarrow M_y = 2\times\left[50{,}4\times(18{,}074\times10^{-3}) + 256\cdot\left(\tfrac{2}{3}\cdot16\times10^{-3}\right)\right] \Rightarrow \boxed{M_y = 7.283\,\text{kN-m}} \rightsquigarrow \varphi_y = \frac{\varepsilon_y}{y} = \frac{5\times10^{-3}}{20\times10^{-3}} \Rightarrow \boxed{\varphi_y = 0{,}250/m}$$

Momento de Cedencia en el acero:



$$\Rightarrow M = 2\cdot\left[56\cdot(18\times10^{-3}) + 432\cdot(10.67\times10^{-3})\right] \Rightarrow \boxed{M = 11.232\ \text{KN-m}} \sim \varphi = \varepsilon / y = \frac{6.75\times10^{-3}}{16\times10^{-3}} \Rightarrow \boxed{\varphi = 421.875\times10^{-3}/m}$$

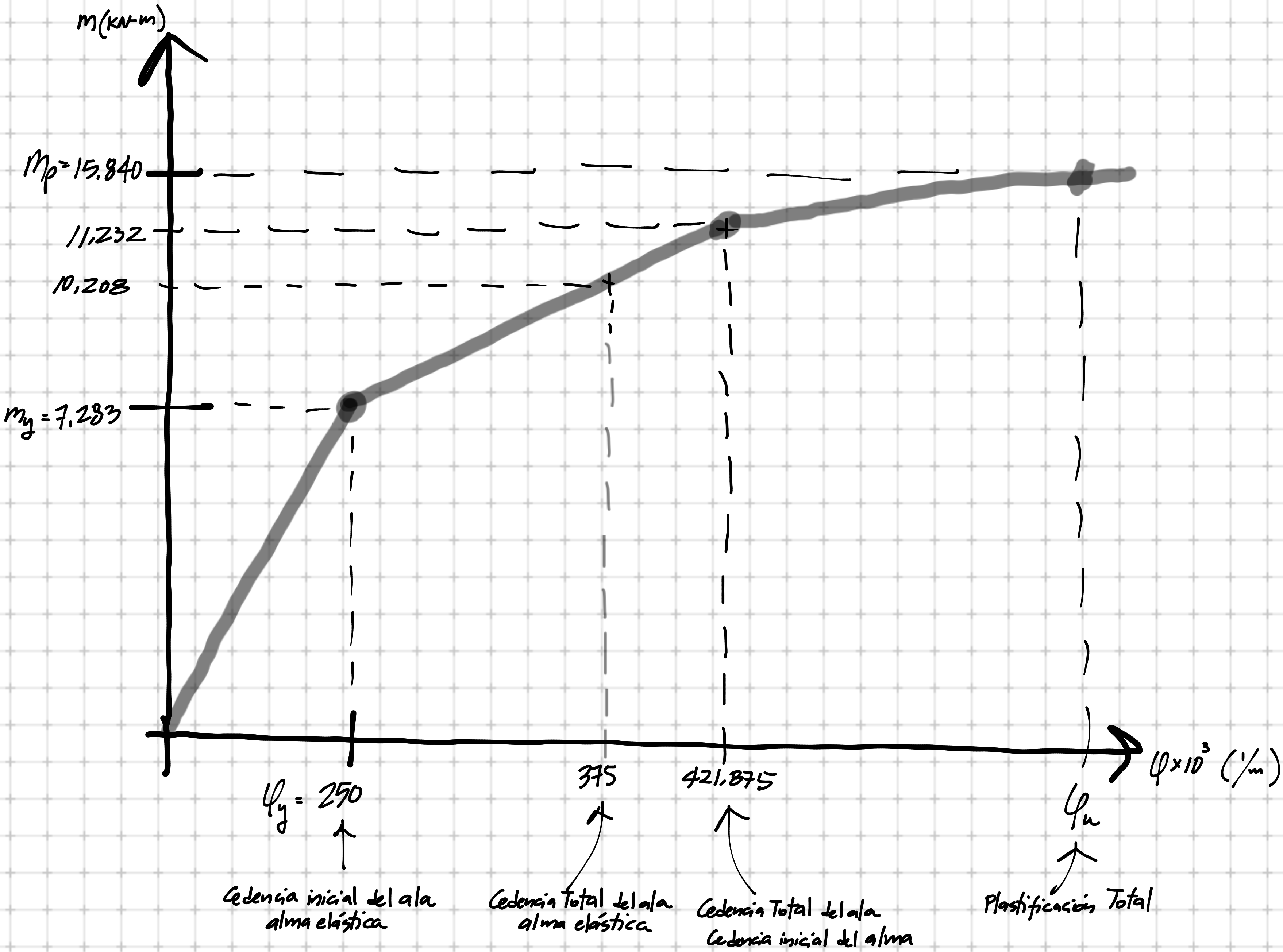
Momento Plástico (M_p):



$$\Rightarrow M_p = 2\cdot\left[56\cdot(18\times10^{-3}) + 864\cdot(8\times10^{-3})\right] \Rightarrow \boxed{M_p = 15.840\ \text{KN-m}} \sim \varphi_u \uparrow\uparrow$$

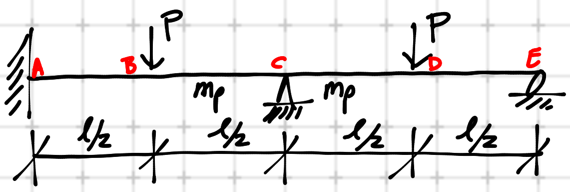
$$\Rightarrow f = M_p / M_y = 15.840/7.283 \rightarrow \boxed{f = 2.175}$$

Gráfica Momento-Curvatura $(M-\varphi)$:

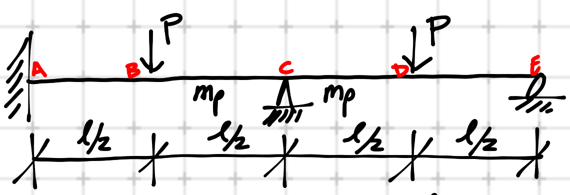


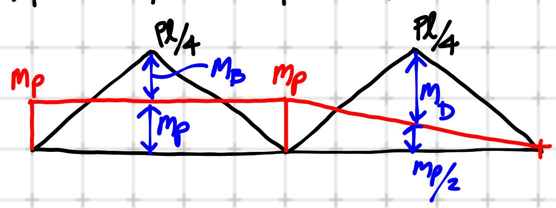
21/Abr/2014

Análisis Plástico (Viga Continua)



Método de Equilibrio





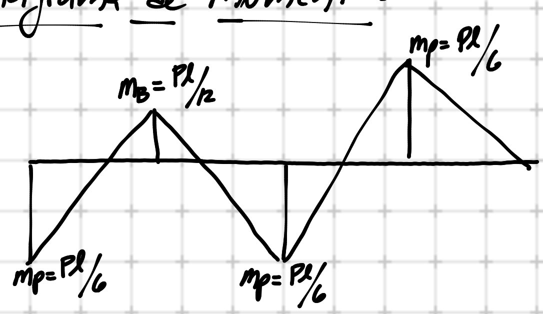
$\rightarrow$ Si $M_B = M_p \rightarrow \quad m_p + m_p = Pl/4 \Rightarrow P = 8m_p/l$

Si $M_D = m_p \rightarrow \quad m_p/2 + m_p = Pl/4 \Rightarrow \boxed{P = 6m_p/l}$ ← Carga de Falla

$\hookrightarrow \boxed{m_p = Pl/6}$

$\Rightarrow \overset{Pl/6}{M_p} + M_B = Pl/4 \rightsquigarrow M_B = Pl/12 < m_p \rightarrow$ ok

Diagrama de Momento:



Fuerzas y Momentos en los Extremos de los elementos:

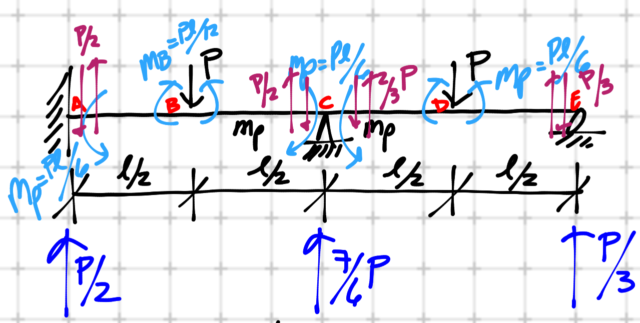
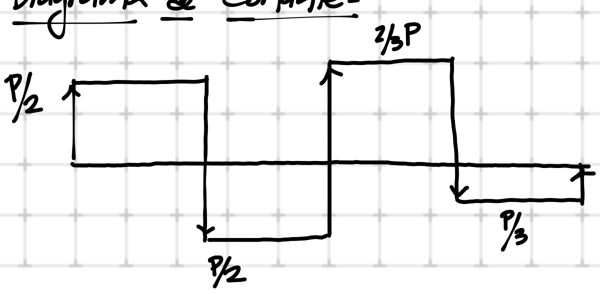
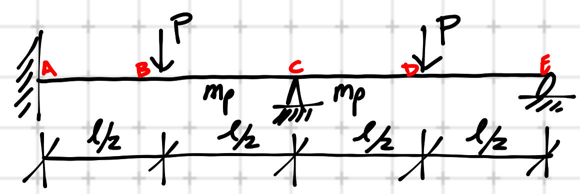


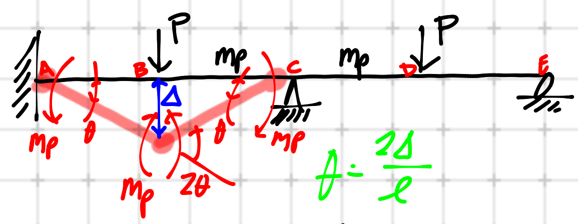
Diagrama de Cortante:



Método Cinemático (Trabajo Virtual):



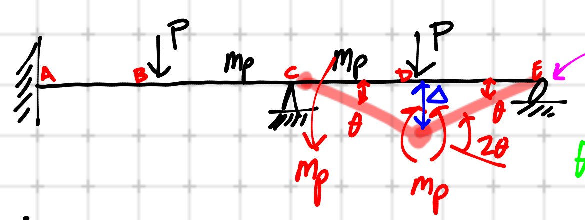
Mecanismo en el Tramo ABC:



$\theta = \frac{2\Delta}{l}$

$P\Delta = M_p\theta + m_p(2\theta) + M_p\theta = 4M_p\theta = 4M_p\left(\frac{2\Delta}{l}\right) = 8m_p/l$

Mecanismo en el Tramo CDE:



Ojo: articulación real $(m=0)$

$\theta = \frac{2\Delta}{l}$

$P \cdot \Delta = M_p\theta + m_p(2\theta) = 3m_p\theta = 3m_p(2\Delta/l)$

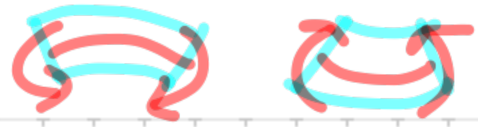
$\hookrightarrow \boxed{P = 6M_p/l}$

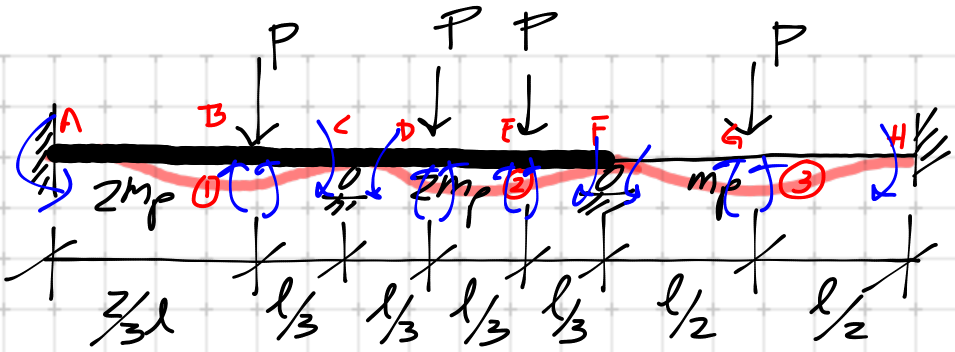
$\hookrightarrow \boxed{M_p = Pl/6}$

El mecanismo real de falla ocurre con la menor carga de colapso.

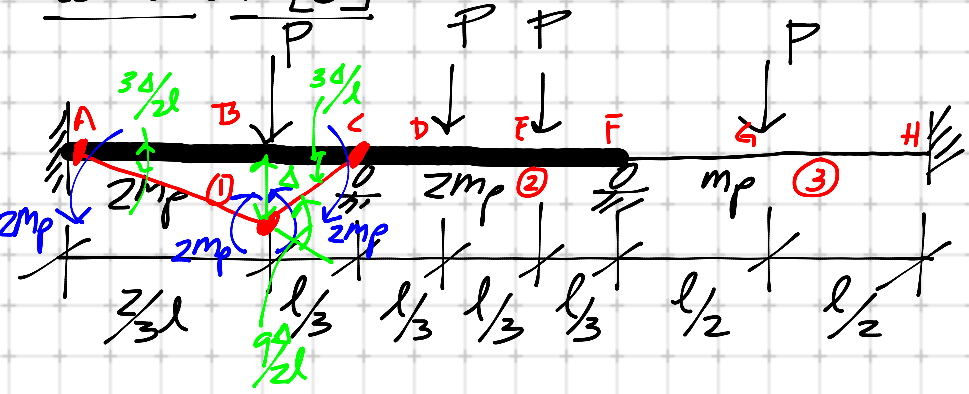
Análisis Plástico (Viga Continua)

23/Abr/2014





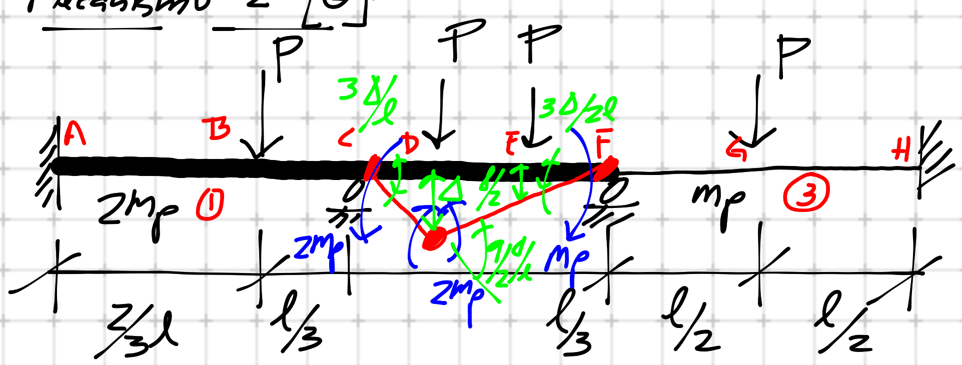
Mecanismo 1 [①]:



$$P\Delta = 2m_p\frac{3\Delta}{2\ell} + 2m_p\frac{9\Delta}{2\ell} + 2m_p\frac{3\Delta}{\ell}$$

$$P = 18\,m_p/\ell \quad \times$$

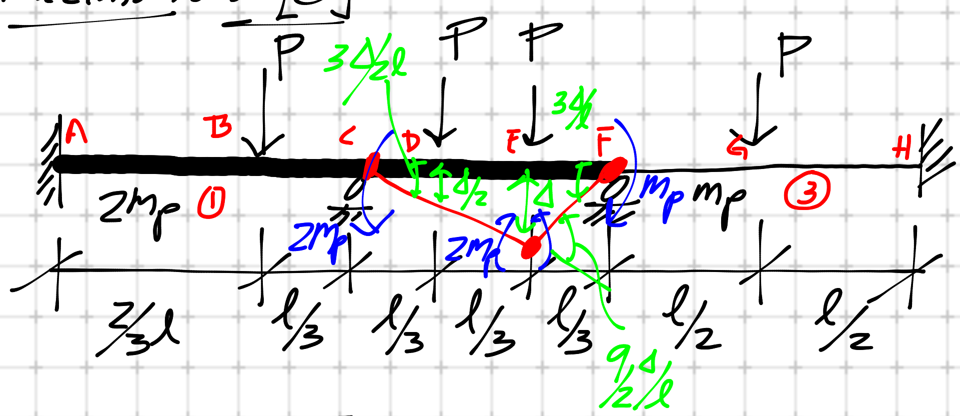
Mecanismo 2 [②]:



$$P\Delta + P\frac{\Delta}{2} = 2m_p\cdot\frac{3\Delta}{\ell} + 2m_p\frac{9\Delta}{2\ell} + m_p\frac{3\Delta}{2\ell}$$

$$P = 11\,m_p/\ell \quad \times$$

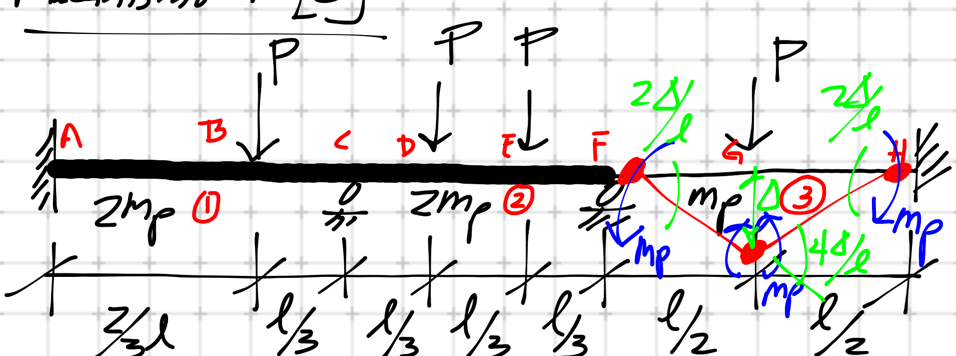
Mecanismo 3 [②]:



$$P\frac{\Delta}{2} + P\Delta = 2m_p\frac{3\Delta}{2\ell} + 2m_p\frac{9\Delta}{2\ell} + m_p\frac{3\Delta}{\ell}$$

$$P = 10\,m_p/\ell \quad \times$$

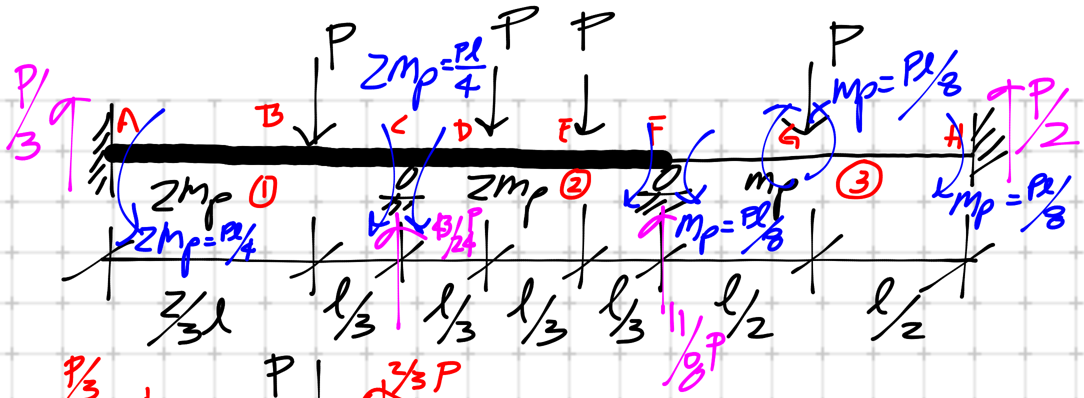
Mecanismo 4 [③]:

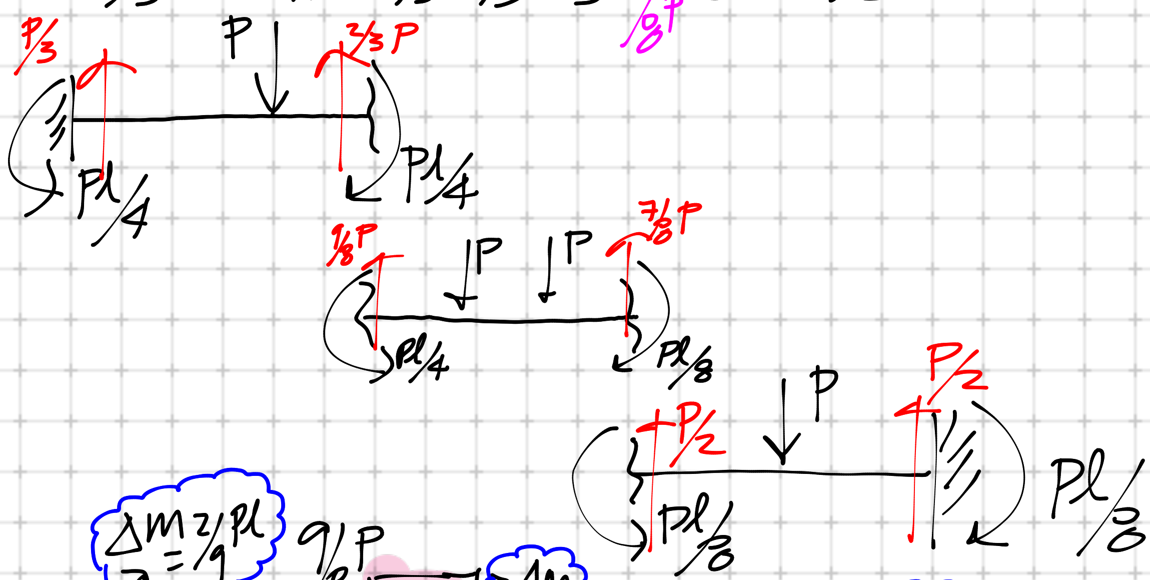


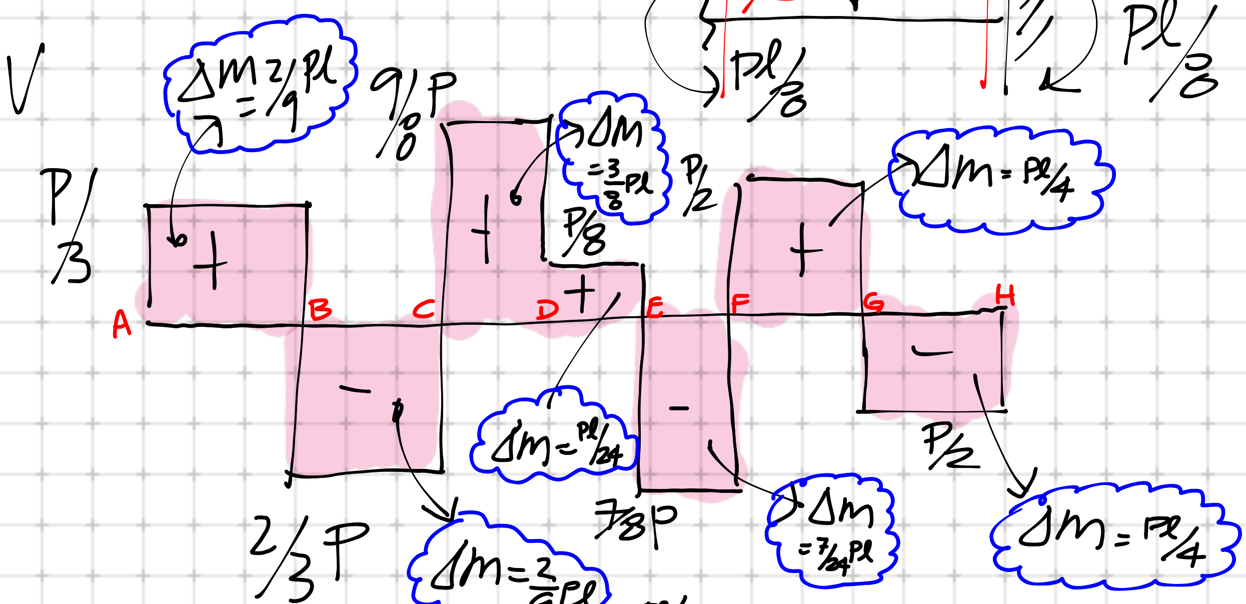
$$P\Delta = m_p\frac{2\Delta}{\ell} + m_p\frac{4\Delta}{\ell} + m_p\frac{2\Delta}{\ell}$$

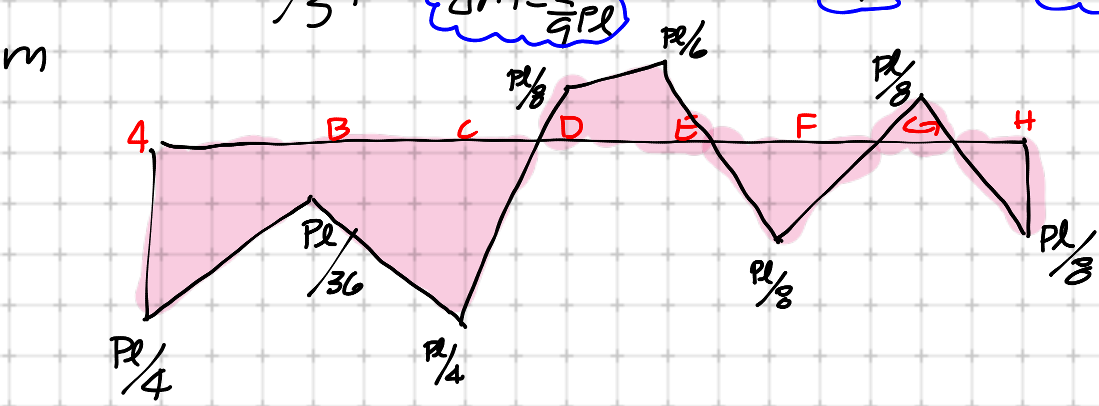
$$\boxed{P = 8\,m_p/\ell} \rightsquigarrow \boxed{m_p = P\ell/8}$$

✓

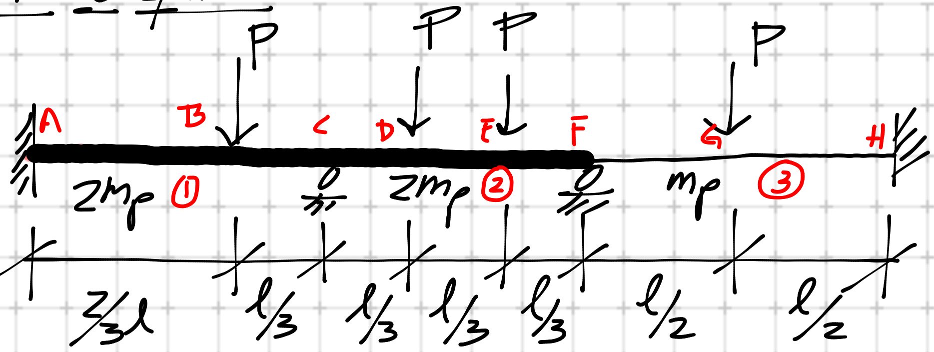

P/3
P
2Mp = Pl/4
P
P
P
Mp = Pl/8
P/2
A B C D E F G H
2Mp ①
2Mp ②
Mp ③
2Mp = Pl/4
13/24 P
Mp = Pl/8
Mp = Pl/8
2/3 l
l/3
l/3
l/3
l/3
l/2
l/2
11/8 P

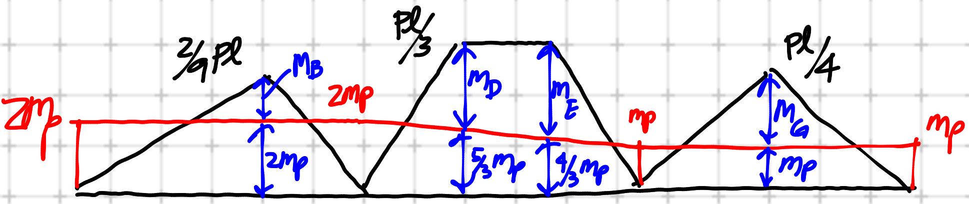

P/3
P
2/3 P
Pl/4
Pl/4
1/8 P
P
P
7/8 P
Pl/4
Pl/8
P
P/2
P/2
Pl/8
Pl/8


V
Δm = 2/9 Pl
9/8 P
Δm = 3/8 Pl
P/3
P/8
P/2
Δm = Pl/4
A B C D E F G H
Δm = Pl/24
2/3 P
Δm = 2/9 Pl
7/8 P
Δm = 7/24 Pl
P/2
Δm = Pl/4


m
Pl/6
Pl/8
Pl/8
A B C D E F G H
Pl/36
Pl/8
Pl/8
Pl/4
Pl/4

Método de Equilibrio





Si $M_B = 2M_p \to 2M_p + 2M_p = \frac{2}{9}Pl \rightsquigarrow P = 18\,M_p/l$

Si $M_D = 2M_p \to 2M_p + \frac{5}{3}M_p = Pl/3 \rightsquigarrow P = 11\,M_p/l$

Si $M_E = 2M_p \to 2M_p + \frac{4}{3}M_p = Pl/3 \rightsquigarrow P = 10\,M_p/l$

Si $M_G = M_p \to M_p + M_p = Pl/4 \rightsquigarrow \boxed{P = 8M_p/l} \rightsquigarrow \boxed{M_p = Pl/8}$

$\to M_B = \frac{2}{9}Pl - 2\overset{Pl/8}{M_p} = -Pl/36 < \overset{Pl/4}{2M_p} \to$ ok

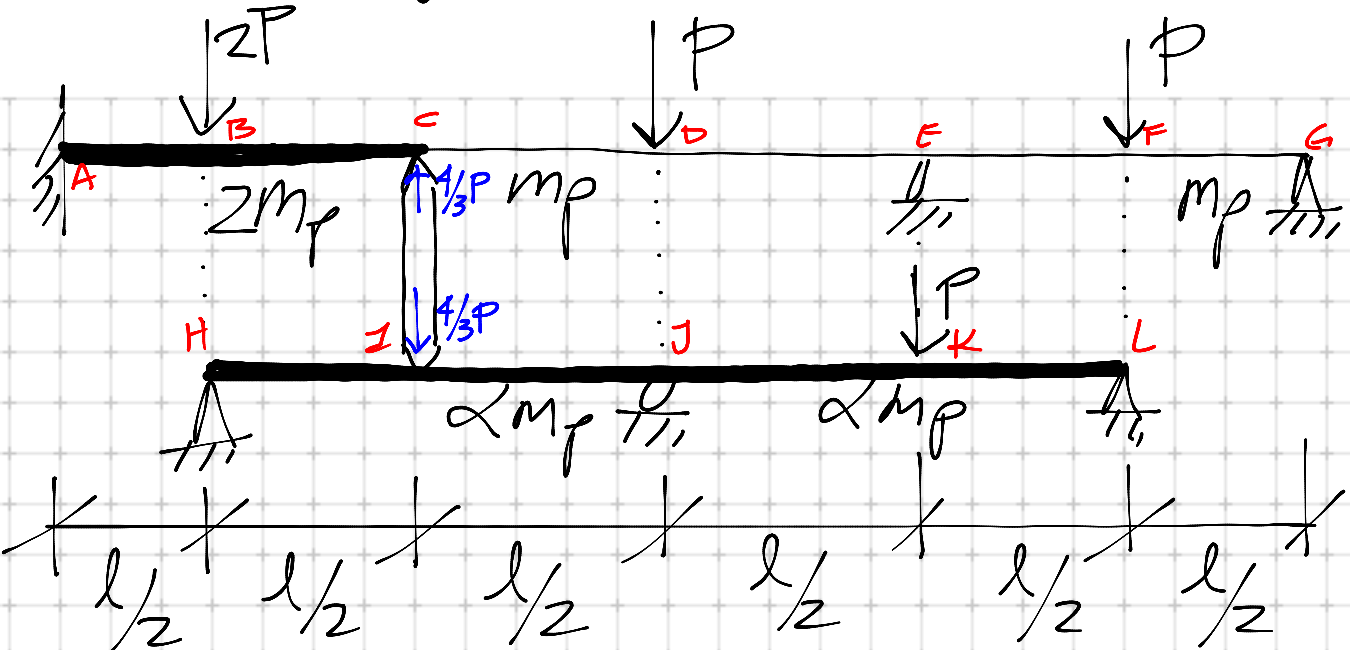
$M_D = Pl/3 - \frac{5}{3}\overset{Pl/8}{M_p} = Pl/8 < 2M_p \to$ ok

$M_E = Pl/3 - \frac{4}{3}\overset{Pl/6}{M_p} = Pl/6 < 2M_p \to$ ok

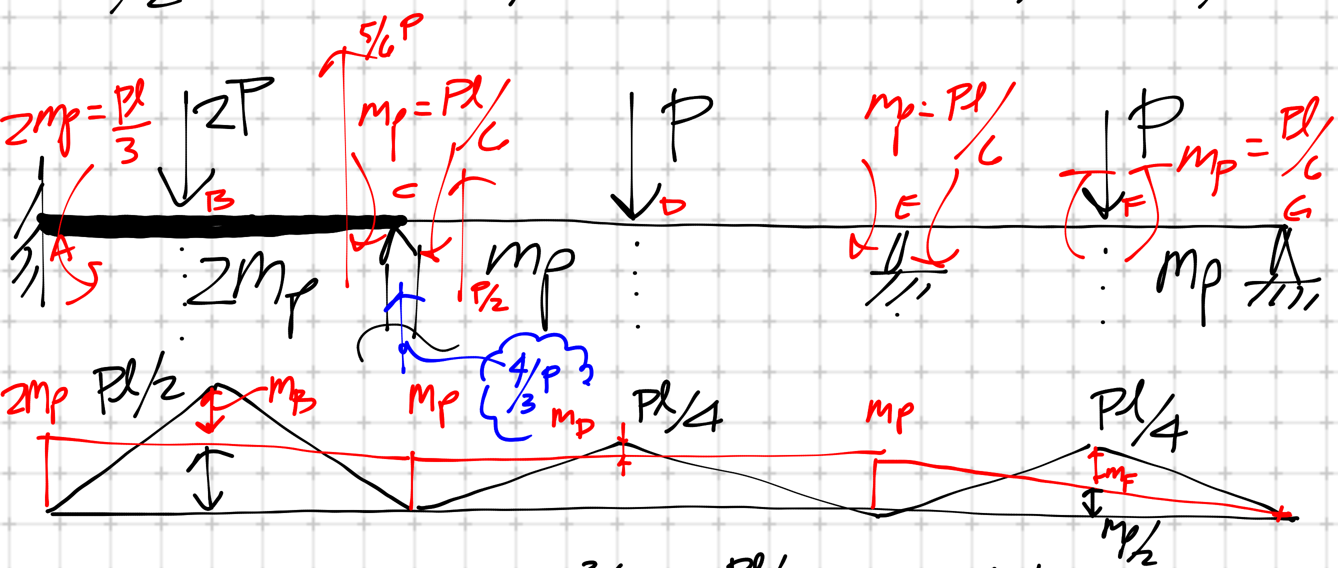
Continua igual que en el método cinemático

Análisis Plástico (Viga Continua)

28/Abr/2014

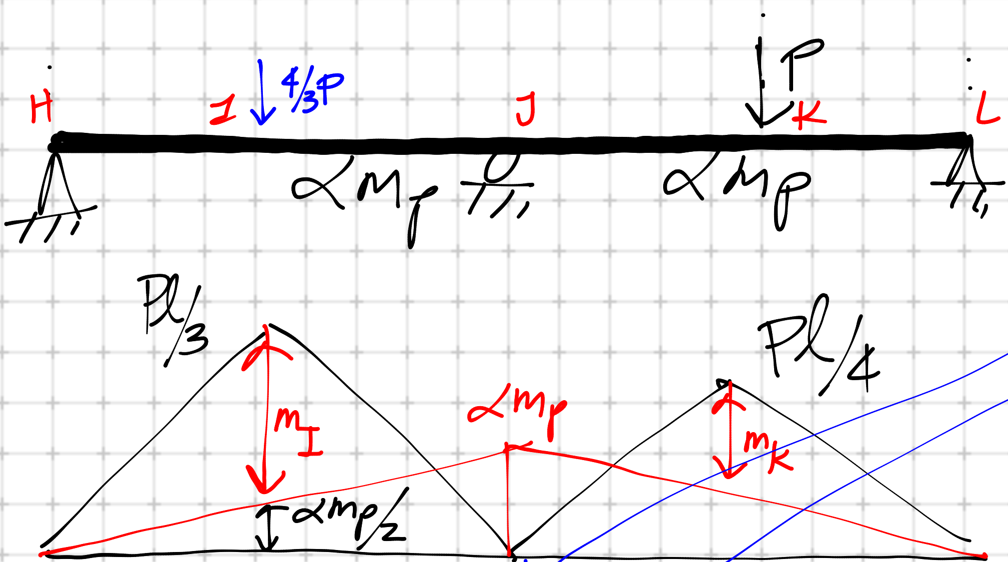


Determine α para que el mecanismo de falla ocurra simultáneamente tanto en la viga superior como inferior.



Si $M_B = 2M_p \rightarrow 2M_p + \frac{3}{2}M_p = Pl/2 \rightsquigarrow P = 7M_p/l$

Si $M_F = M_p \rightarrow M_p + M_p/2 = Pl/4 \rightarrow P = 6M_p/l \rightarrow \boxed{M_p = Pl/6}$

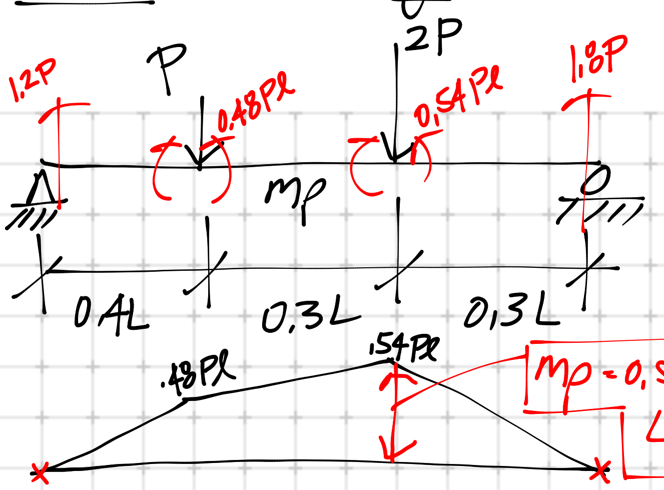


Si $M_I = \alpha M_p \rightarrow \alpha M_p + \alpha M_p/2 = Pl/3$

$\frac{3}{2}\alpha\left(Pl/6\right) = Pl/3 \rightsquigarrow \boxed{\alpha = 4/3}$

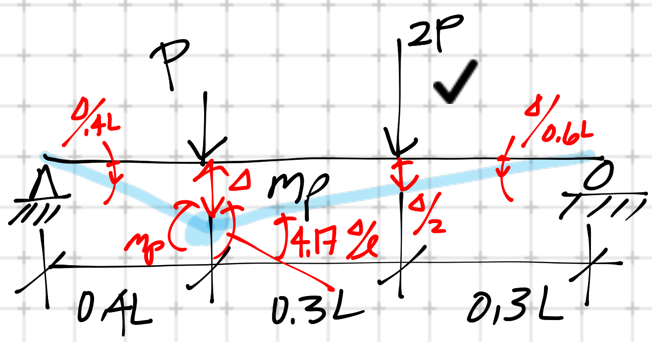
28/Abr/2014

Análisis Plástico (Viga Determinada)

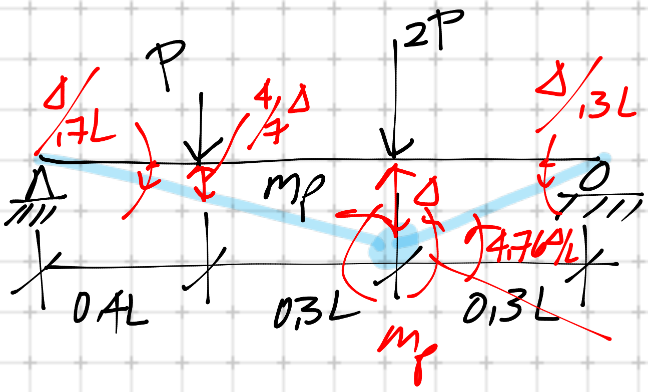


$M_p = 0.54PL$

$\hookrightarrow P = 1.85\, M_p/L$



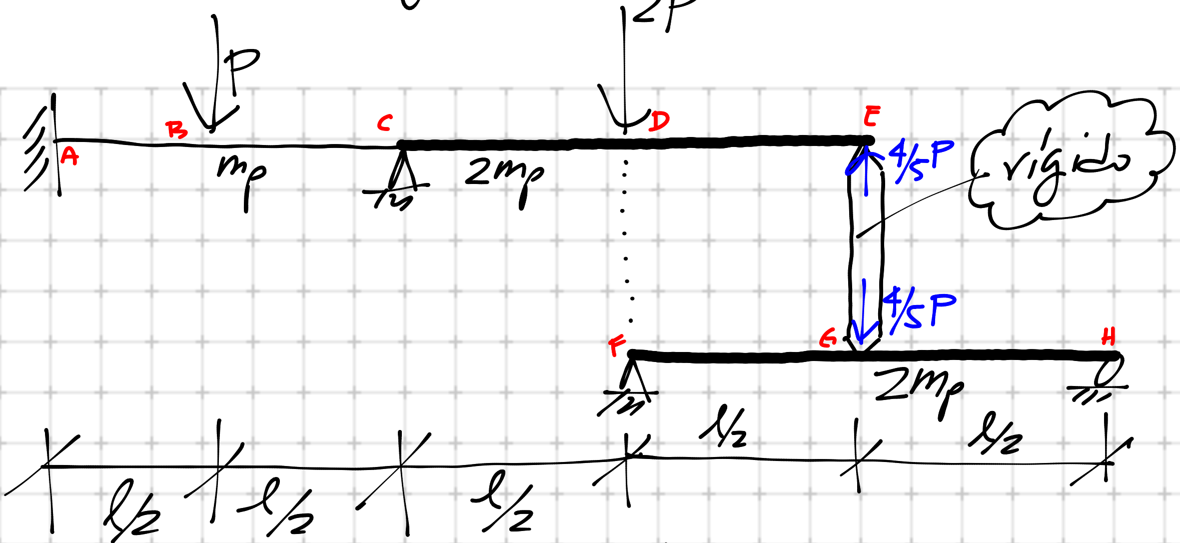
$$P\cancel{\Delta} + 2P \cdot \frac{\cancel{\Delta}}{2} = M_p\left(4.166667\, \frac{\cancel{\Delta}}{L}\right) \rightsquigarrow P = 2.08\, M_p/L$$

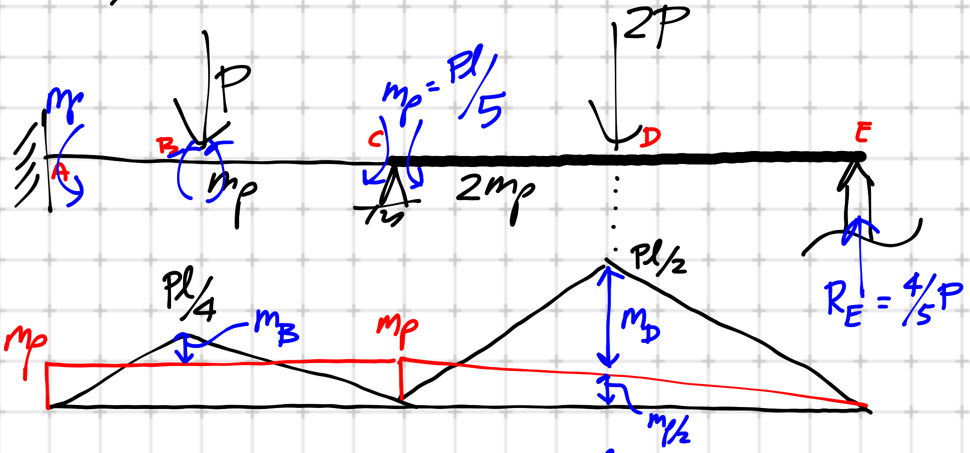


$$P \cdot \frac{4}{7}\Delta + 2P\Delta = M_p\left(4.76\, \frac{\Delta}{L}\right) \rightsquigarrow \boxed{P = 1.85\, M_p/L}$$

Análisis Plástico (Viga Continua)

28/Abr/2014

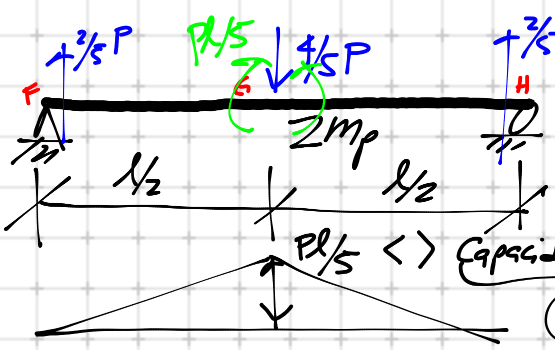




Si $m_B = m_p \to m_p + m_p = \frac{Pl}{4} \leadsto P = 8m_p/l$ ×

Si $m_D = 2m_p \to m_p/2 + 2m_p = \frac{Pl}{2} \leadsto \boxed{P = 5m_p/l} \leadsto \boxed{m_p = \frac{Pl}{5}}$

$m_B = \frac{Pl}{4} - m_p = \frac{Pl}{4} - \frac{Pl}{5} = \frac{Pl}{20} < m_p \to OK$



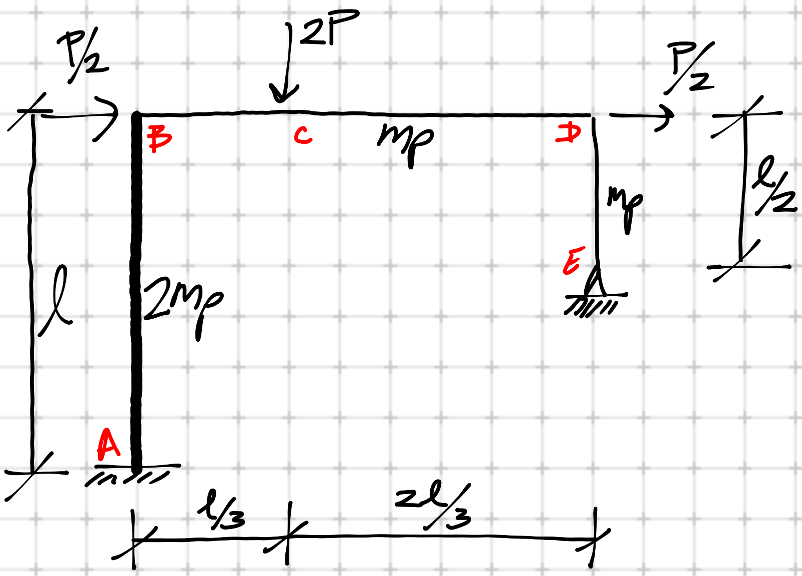
Pl/5 <> Capacidad de la viga

$2m_p = 2 \cdot \frac{Pl}{5} = \frac{2Pl}{5}$

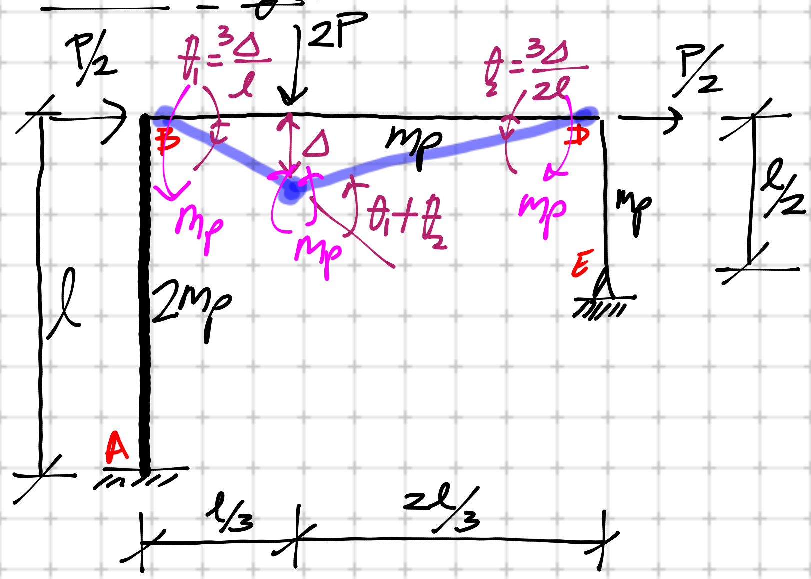
$\frac{Pl}{5} < 2m_p \to OK$

30/Abr

Análisis Plástico [Marcos]

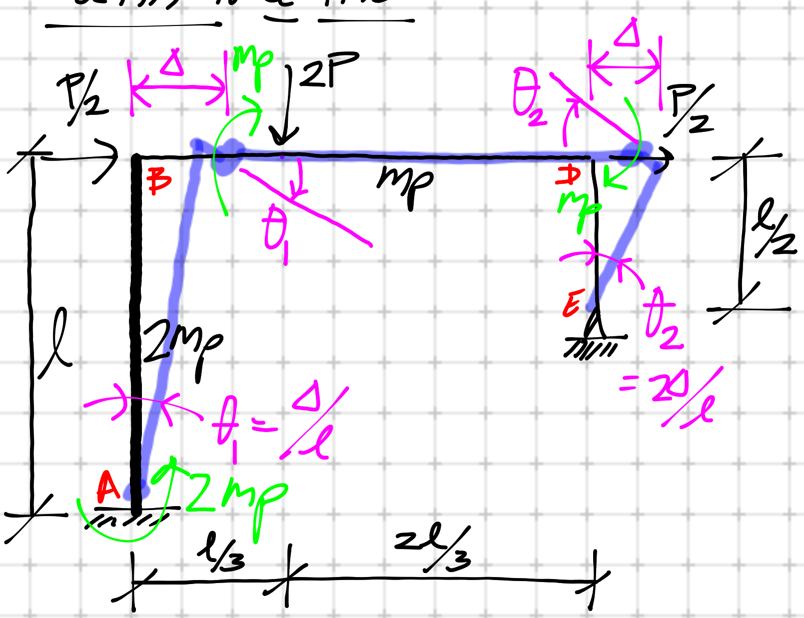


Mecanismo de Viga:



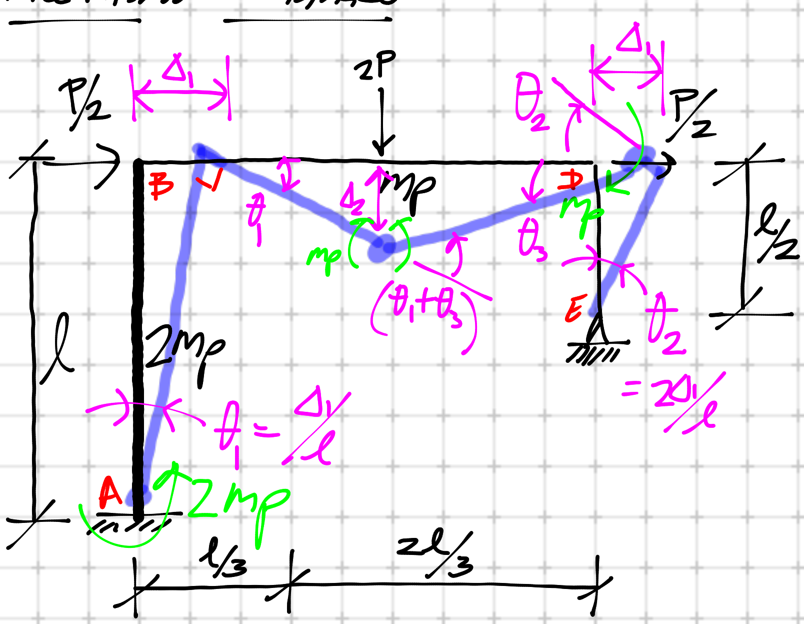
$$2P\cancel{\Delta} = M_p\left(3\Delta/\ell\right) + M_p\left(\tfrac{3}{2}\Delta/\ell + 3\Delta/\ell\right) + M_p \cdot 3\Delta/2\ell \rightarrow P = \tfrac{9}{2}\, M_p/\ell \checkmark$$

Mecanismo de Panel



$$\left[\tfrac{P}{2}\cancel{\Delta}\right]2 = 2M_p\,\cancel{\Delta}/\ell + M_p\,\cancel{\Delta}/\ell + M_p\,2\cancel{\Delta}/\ell \rightsquigarrow P = 5\,M_p/\ell \times$$

Mecanismo Combinado

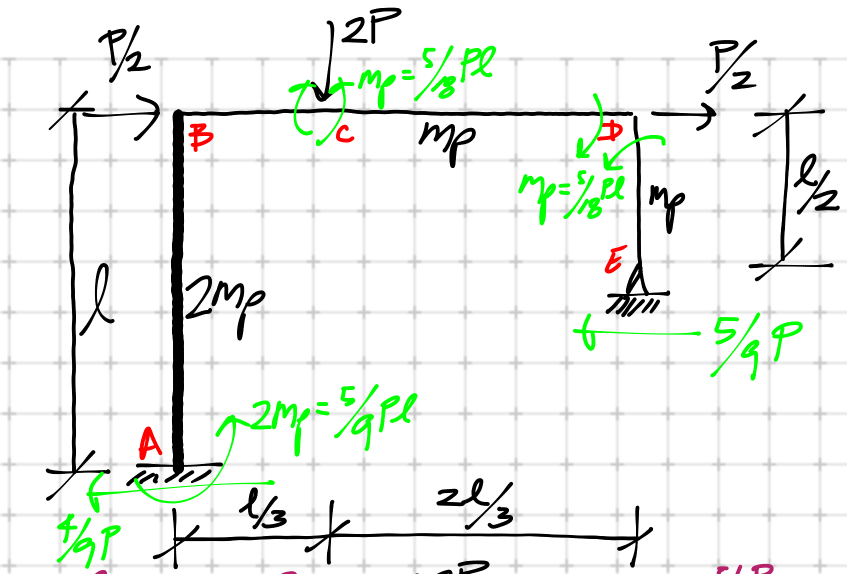


$$\left[\tfrac{P}{2}\cdot\cancel{\Delta_1}\right]\cdot 2 + 2P\cdot\cancel{\Delta_1}/3 = 2M_p\frac{\cancel{\Delta_1}}{\ell} + M_p\left(\cancel{\Delta_1}/\ell + \cancel{\Delta_1}/2\ell\right) + M_p\left(\cancel{\Delta_1}/2\ell + 2\cancel{\Delta_1}/\ell\right)$$

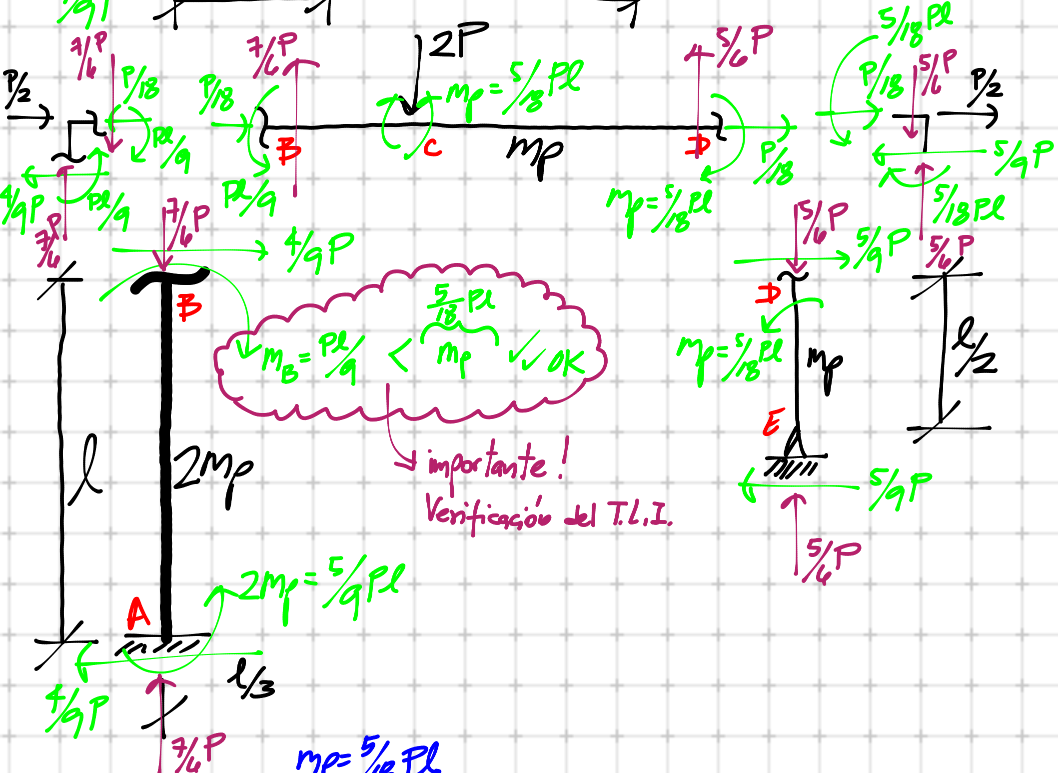
$$\hookrightarrow \boxed{P = \tfrac{18}{5}\, M_p/\ell}\ ; \quad \boxed{M_p = \tfrac{5}{18}\, P\ell}$$

$$\Delta_2 = \theta_1 \cdot \ell/3 = \Delta_1/\cancel{\ell} \cdot \cancel{\ell}/3 = \Delta_1/3$$

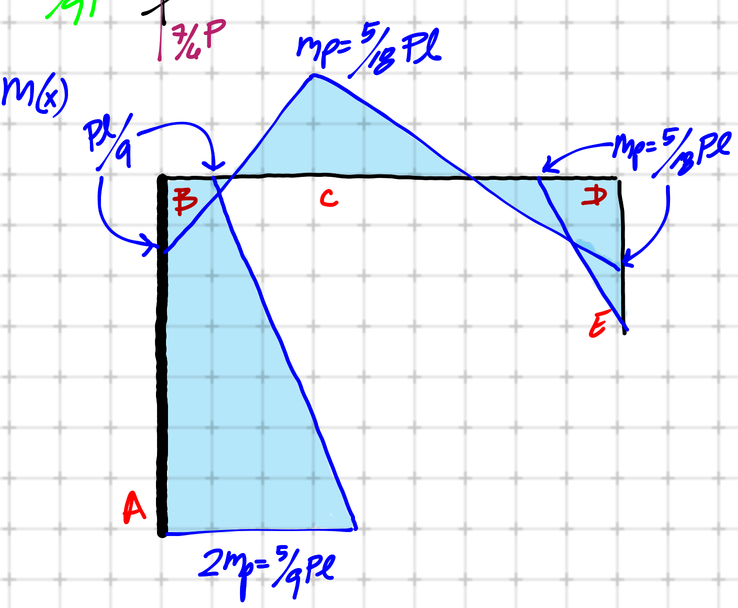
$$\theta_3 = \Delta_2 \cdot \frac{3}{2\ell} = \Delta_1/\cancel{3} \cdot \frac{\cancel{3}}{2\ell} = \Delta_1/2\ell$$



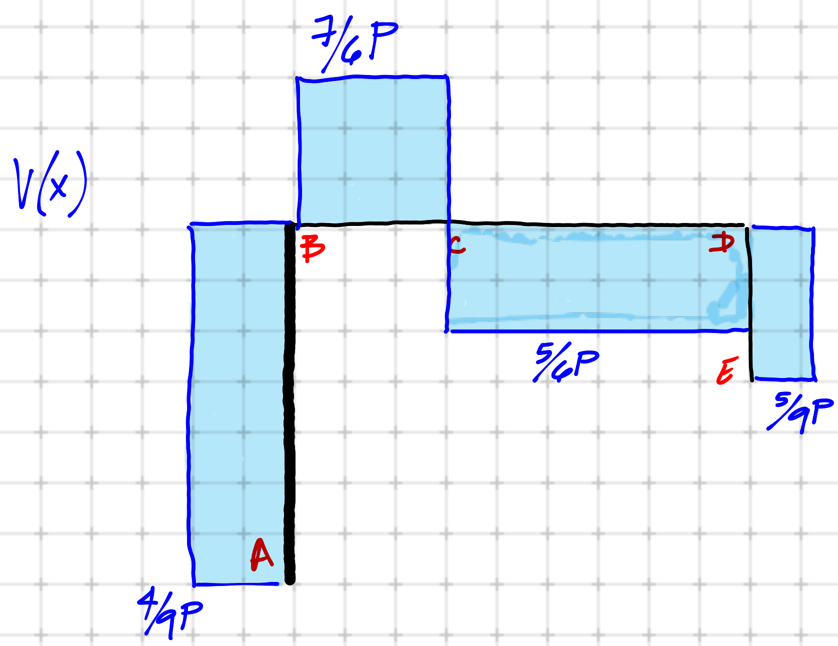
P/2
2P
$m_p = 5/18\,Pl$
B
C
m_p
D
P/2
$l/2$
$m_p = 5/18\,Pl$
m_p
E
5/9 P
l
$2m_p$
$2m_p = 5/9\,Pl$
A
4/9 P
$l/3$
$2l/3$



2P
$m_p = 5/18\,Pl$
7/6 P
P/18
P/2
Pl/9
7/6 P
B
C
m_p
5/6 P
D
5/18 Pl
P/18
5/6 P
P/2
5/9 P
$m_p = 5/18\,Pl$
5/18 Pl
4/9 P
7/6 P
4/9 P
5/6 P
5/9 P
5/6 P
B
$M_B = Pl/9 < \overset{5/18\,Pl}{m_p}$ ✓ OK
importante!
Verificación del T.L.I.
D
$m_p = 5/18\,Pl$
m_p
E
$l/2$
l
$2m_p$
5/9 P
5/6 P
$2m_p = 5/9\,Pl$
A
4/9 P
$l/3$
7/6 P



M(x)
$m_p = 5/18\,Pl$
Pl/9
$m_p = 5/18\,Pl$
B
C
D
E
A
$2m_p = 5/9\,Pl$



V(x)
7/6 P
B
C
D
5/6 P
E
5/9 P
A
4/9 P