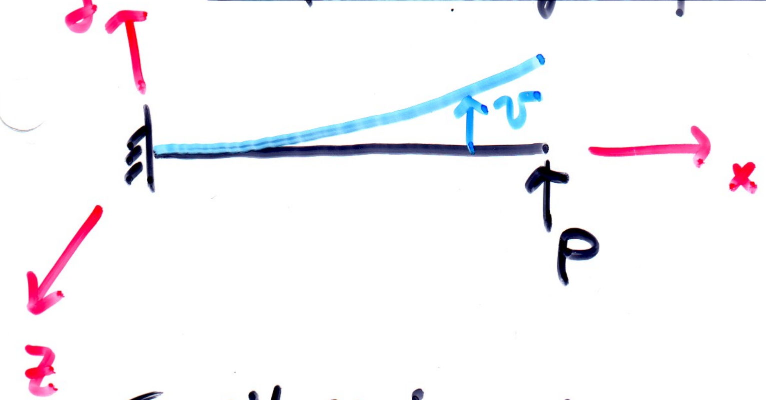
Esfuerzos y Deflexiones en vigas



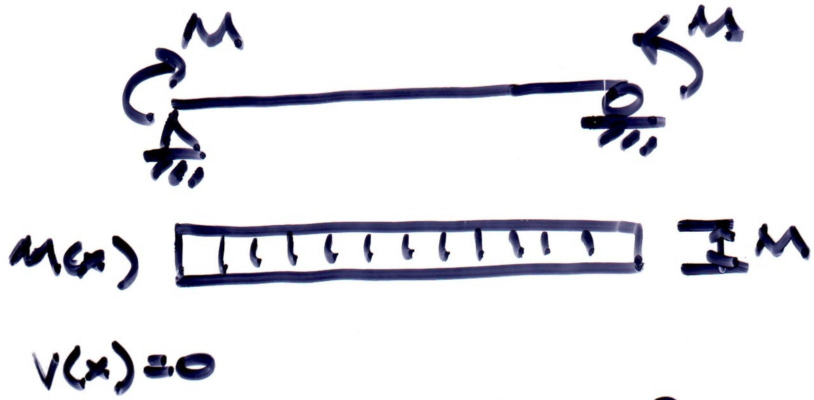
v: deflexión

Consideraciones:

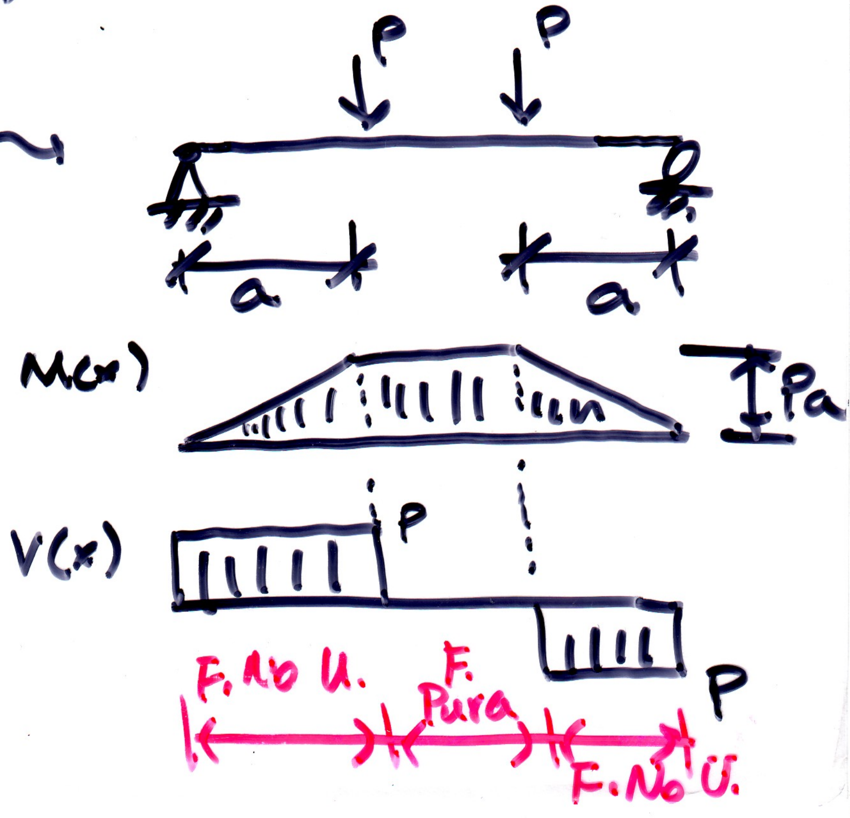
↳ Sección transversal simétrica con respecto al eje y.

↳ La Flexión ocurre en el mismo plano vertical

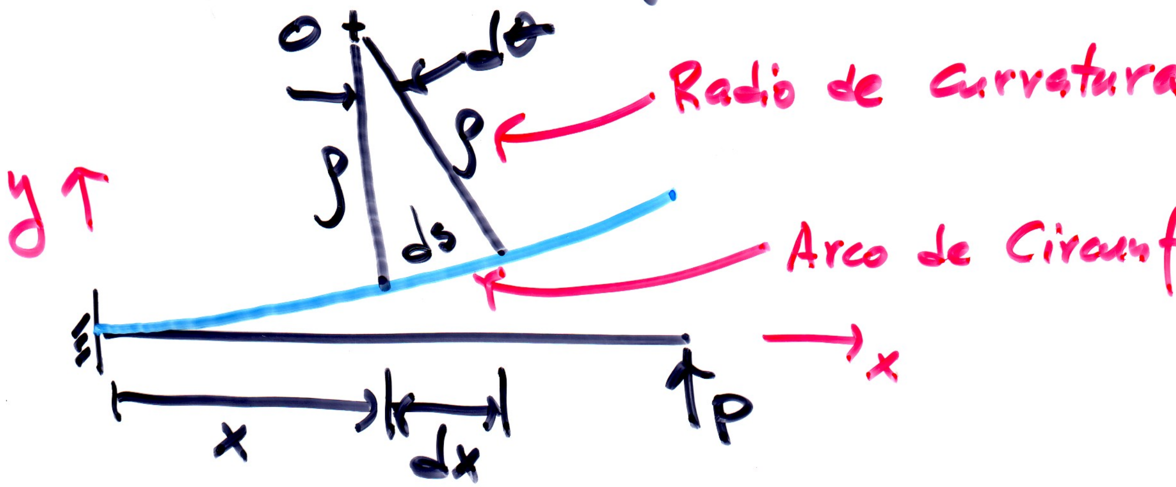
Flexión Pura ~



Flexión No Uniforme ~



Curvatura de una Viga



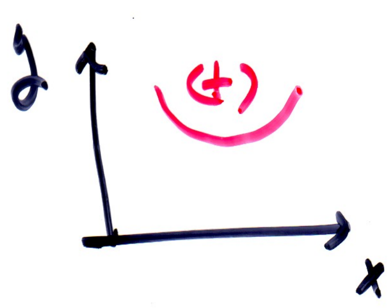
Curvatura ⤳ $k = \frac{1}{\rho}$

Arco de circunferencia ⤳ $ds = \rho\, d\theta$

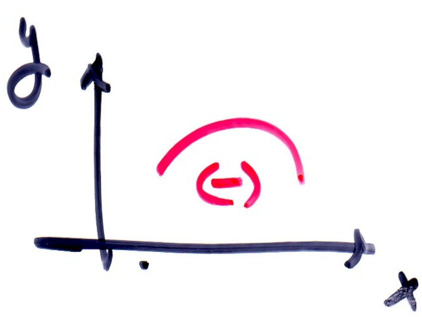
$$k = \frac{1}{\rho} = \frac{d\theta}{ds}$$

Aplicando la teoria de deformaciones pequeñas (la curva deflectada es casi plana): $ds \approx dx$

$$\Rightarrow k = \frac{1}{\rho} = \frac{d\theta}{dx}$$



Curvatura (+)

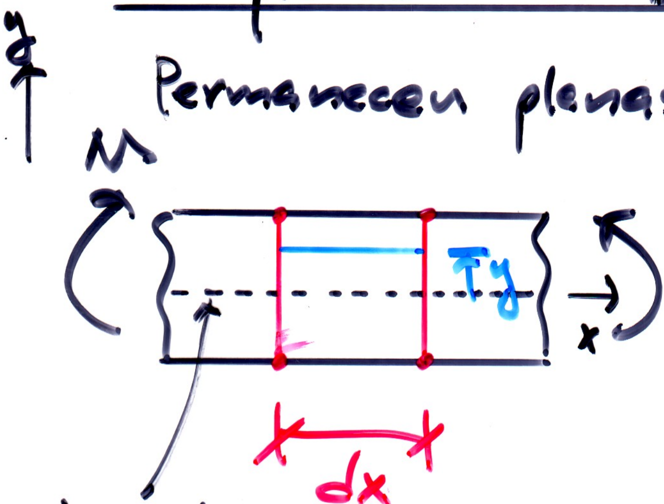


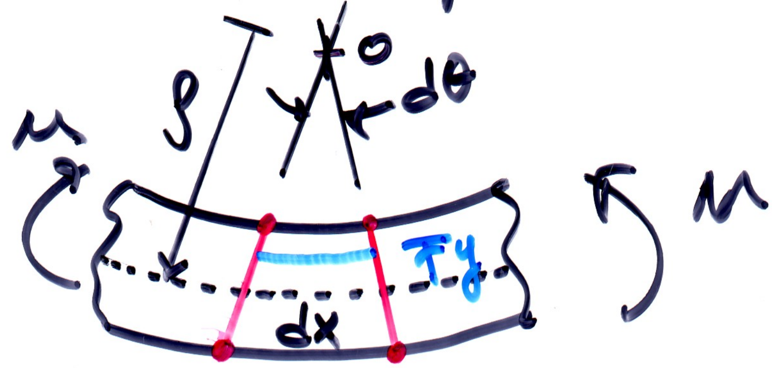
Curvatura (−)

Deformación Unitaria Longitudinal

Principio de Navier - Bernoulli: Secciones planas

Permanecen planas.





eje neutro (e.n.)

Deformacion unitaria de un punto a una distancia "y" del eje neutro:

$$\varepsilon_x = \frac{\Delta L}{L_0} = \frac{L_f - L_0}{L_0}$$

$\hookrightarrow L_f = (\rho - y)\,d\theta = dx - y\,d\theta = dx - y\dfrac{dx}{\rho}$

$\hookrightarrow L_0 = dx$

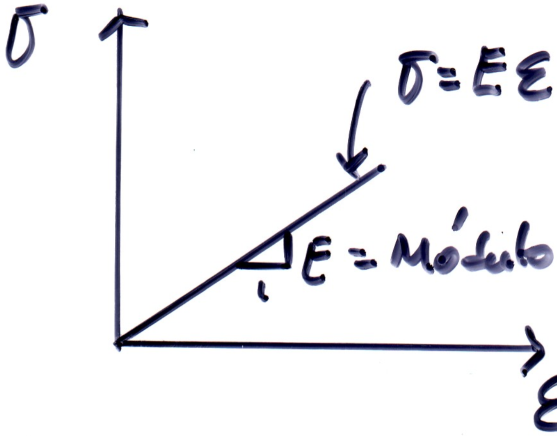
$$\Rightarrow \varepsilon_x = -\frac{y}{\rho} = -Ky$$

$\hookrightarrow \varepsilon_x^{(-)} \rightsquigarrow$ compresión

$\varepsilon_x^{(+)} \to$ tensión

Esfuerzos Normales en Vigas

Ley Constitutiva: Ley que relaciona σ vs ε. En este curso se utilizará la **Ley de Hooke** $\rightsquigarrow$ $\sigma = E\varepsilon$



E = Módulo de Elasticidad

↳ Materiales Linealmente Elásticos.

$$\sigma_x = E\varepsilon_x = -E\frac{y}{\rho} = -KEy$$

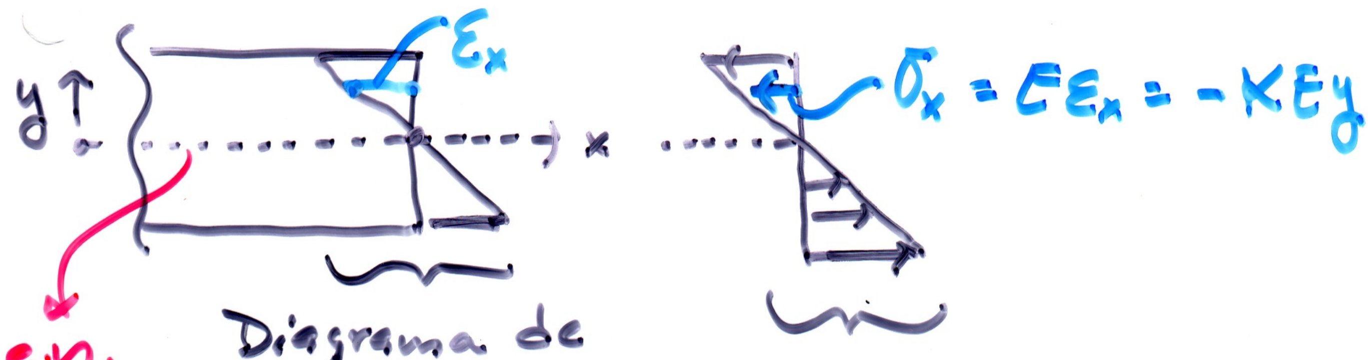


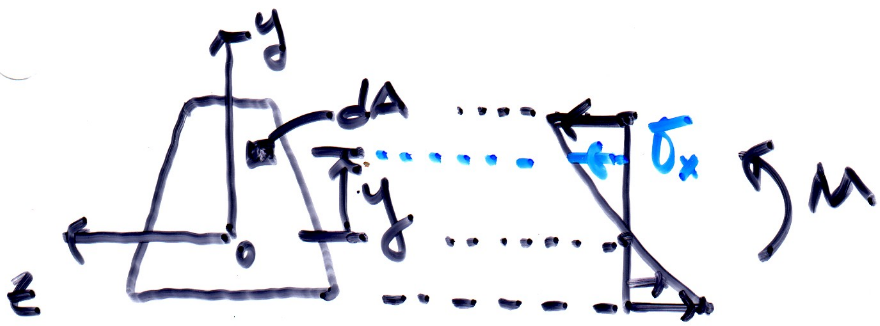
Diagrama de Deformaciones Unitarias

Diagrama de Esfuerzos

$\sigma_x^{(+)} \rightsquigarrow$ Tensión

$\sigma_x^{(-)} \rightsquigarrow$ Compresión

Ubicación del Eje Neutro



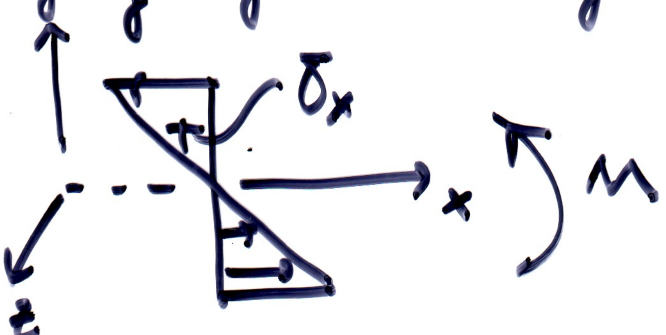
$$\Sigma F_x = 0$$

$$\int_A \sigma_x \, dA = -\int_A K E y \, dA = 0$$

↳ $\int_A y \, dA = 0$

⇒ El eje neutro está localizado en el centroide de la sección transversal.

↳ Válido para materiales linealmente elásticos y vigas con carga axial nula.



$$dM = \overbrace{(-\sigma_x \, dA)}^{dF_x} \, y$$

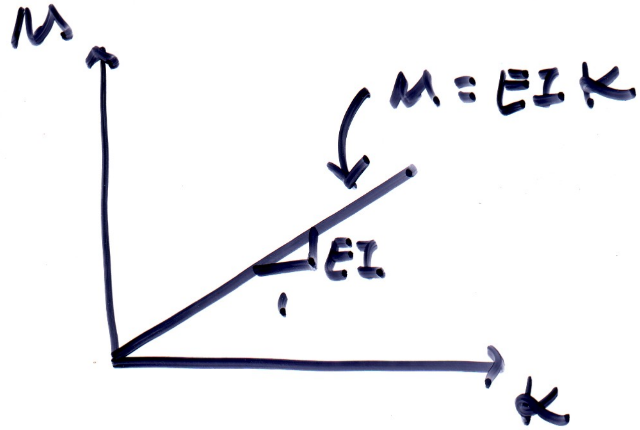
$$\Sigma M_z = M = -\int_A \overset{-KEy}{\sigma_x} \, y \, dA$$

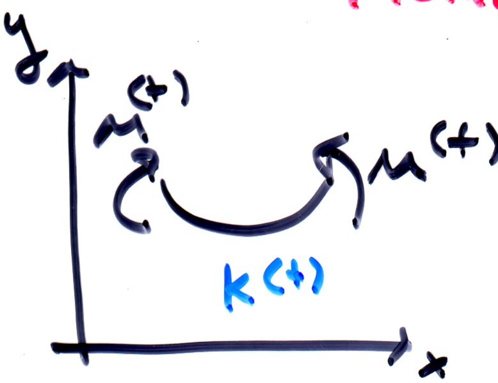
$$M = \int_A K E y^2 \, dA = K E \overbrace{\int_A y^2 \, dA}^{I}$$

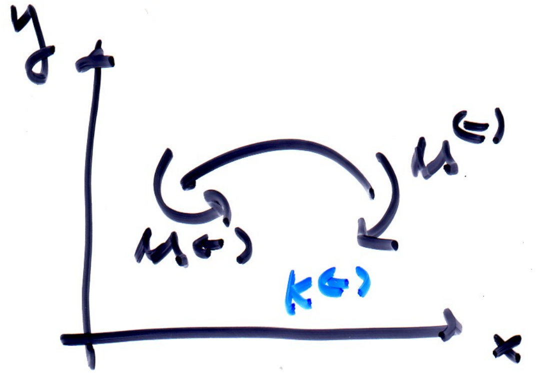
$$\Rightarrow M = KEI$$

$$\Rightarrow K = \frac{1}{\rho} = \frac{M}{EI}$$

Rigidez Flexionante





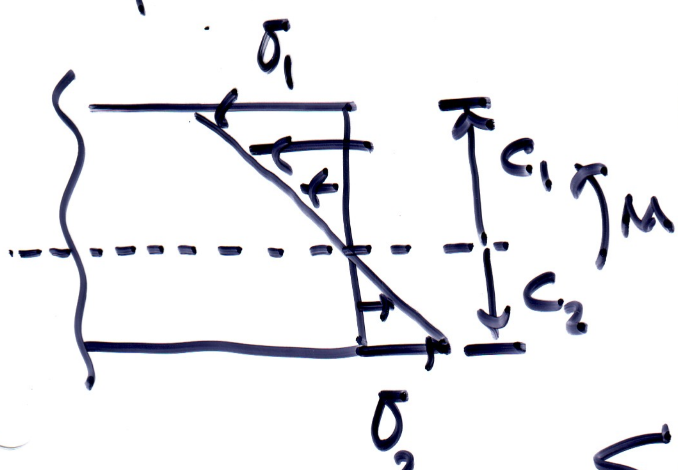


Calculo de Esfuerzos Normales:

$$\sigma_x = -KEy = -\frac{M}{\cancel{E}I}\cancel{E}y \rightsquigarrow \boxed{\sigma_x = -\frac{My}{I}}$$

Esfuerzos de Flexión

Esfuerzos Máximos:

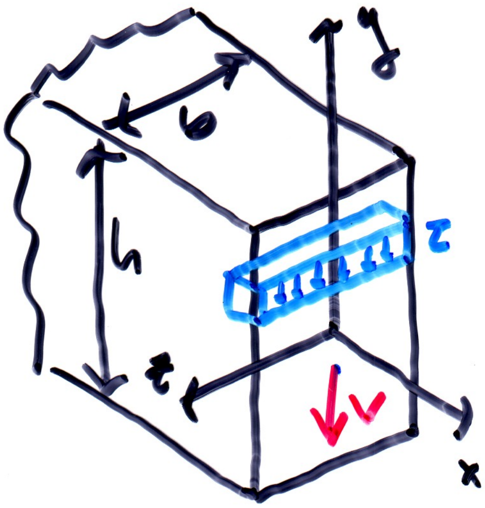


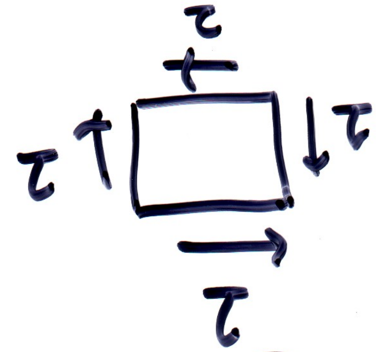
$$\sigma_1 = -\frac{Mc_1}{I} \; ; \; \sigma_2 = \frac{Mc_2}{I}$$

$$S_1 = \frac{I}{c_1} \; ; \; S_2 = \frac{I}{c_2}$$

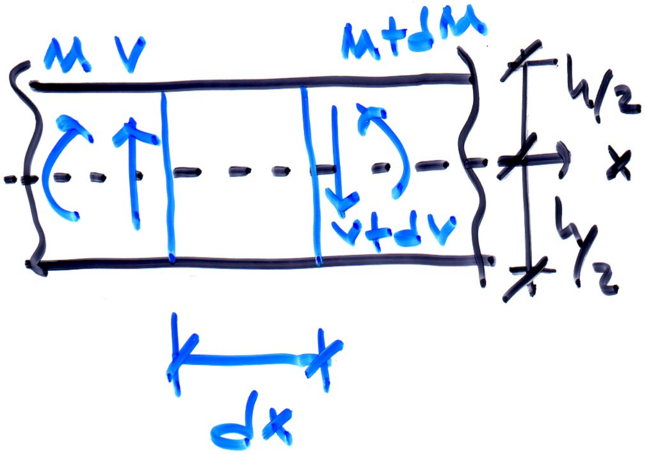
S: Módulo de Sección

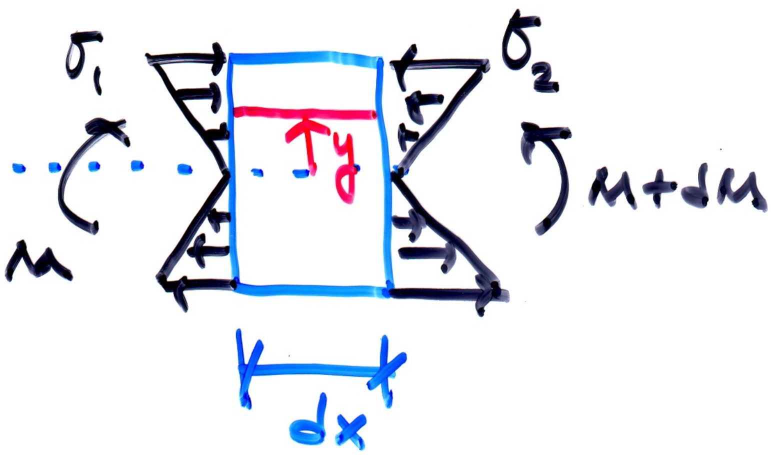
Esfuerzos Cortantes en Vigas Rectangulares

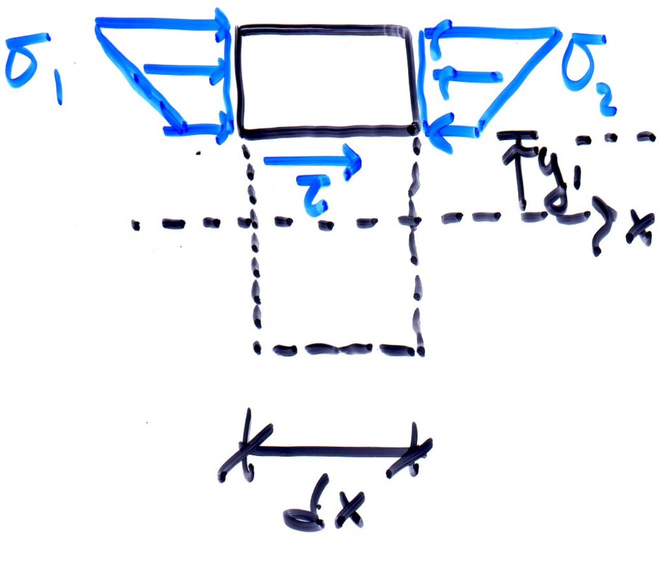




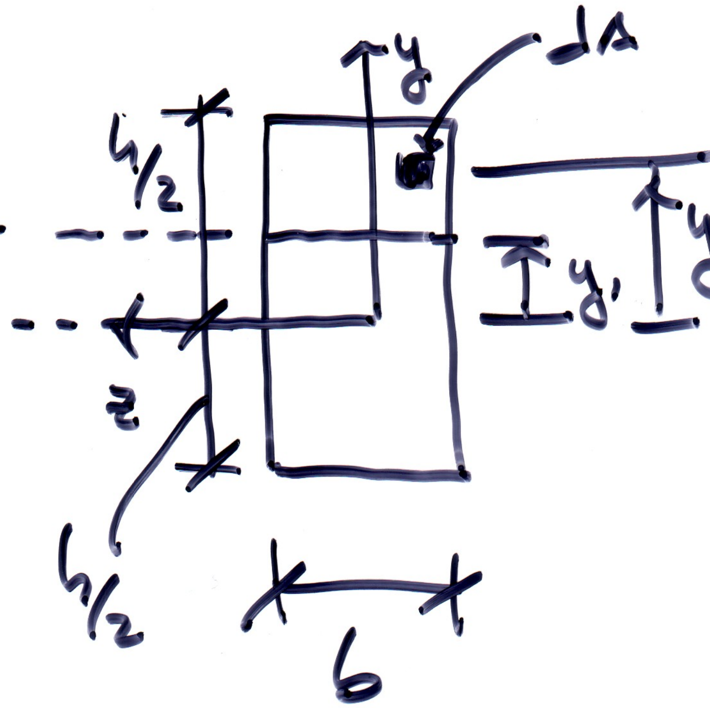
Cortante Puro







Sección Longitudinal



Sección Transversal

$$\sigma_1 = \frac{My}{I} \quad ; \quad \sigma_2 = \frac{(M+dM)y}{I}$$

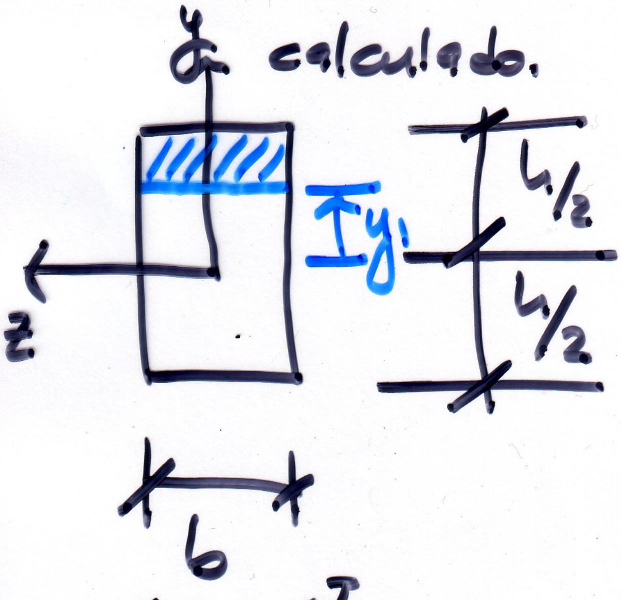
$$\sum F_x = 0$$

$$\int \sigma_1 \, dA + \tau b \, dx - \int \sigma_2 \, dA = 0$$

$$\int \frac{My}{I} dA - \int \frac{(M+dM)}{I} y \, dA + \tau b \, dx = 0$$

$$\tau = \underbrace{\frac{dM}{dx}}_{V}\left(\frac{1}{Ib}\right)\underbrace{\int y \, dA}_{Q} \rightsquigarrow \boxed{\tau = \frac{VQ}{Ib}}$$

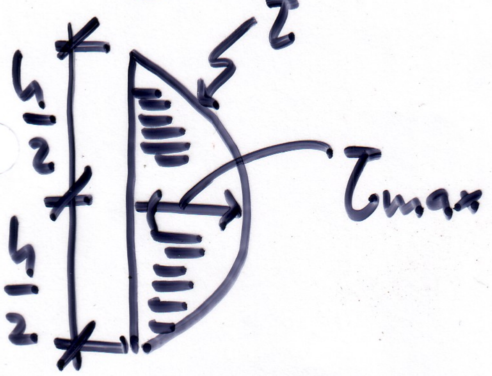
Q : Primer momento del área con respecto al eje neutro al nivel al cual el esfuerzo cortante es calculado.



$$Q = b\left(\frac{h}{2} - y_1\right)\left[\frac{1}{2}\left(y_1 + \frac{h}{2}\right)\right]$$

$$Q = \frac{b}{2}\left(\frac{h^2}{4} - y_1^2\right)$$

$$\Rightarrow \boxed{\tau = \frac{V}{2I}\left(\frac{h^2}{4} - y_1^2\right)}$$



$$\rightsquigarrow \boxed{\tau_{max} = \frac{Vh^2}{8I} = \frac{3}{2}\frac{V}{bh}}$$